FELiCS: Finite Element Linearized (Combustion) Solver

- Addressing flow dynamics using linearized governing flow equations -

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FLOW
INSTABILITIES & DYNAMICS
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The philosophy of FELiCS

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Turbulent flows are driven by coherent structures and are key for engineering applications.

Turbulent production ▪ Scalar mixing and momentum transfer ▪ Aerodynamic noise ▪ Instability and resonance

The features of FELiCS

FELiCS allows solving the linearized governing equations in frequency space for operator-based analysis.

- Spectral analysis of turbulent flows
- Stability analysis and coherent structure prediction
- Sensitivity analysis and adjoint-based optimization

FELiCS is multi-physics.

- Mass and momentum equations (Navier-Stokes)
- Enthalpy equation
- Scalar transport equations
- Combustion models

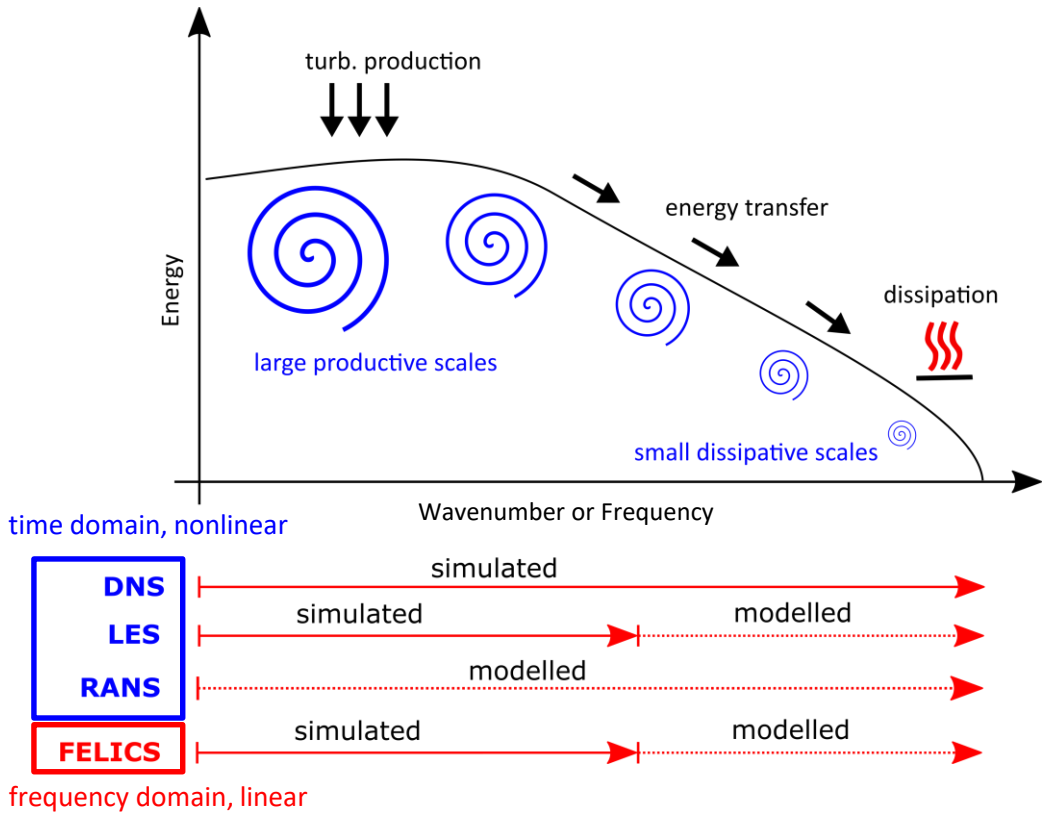
FELiCS complements high-fidelity simulations (CFD) and measurements.

- Mean fields are the only input to FELiCS!
- Computationally fast due to symmetries in the mean field.

FELiCS is versatile.

- Unstructured grids
- Complex geometries
- Interface to CFD solvers (Nek5000, AVBP, OpenFoam, Ansys CFX)
- Various turbulence models
- Cartesian and cylindrical coordinates
- Using FEniCS programming environment

FELiCS compared to conventional CFD



The mathematics behind FELiCS

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Governing equations (e.g. Navier-Stokes)

$$\frac{Du}{Dt} = -\nabla p + \nabla \cdot (\nu \nabla u), \quad \nabla \cdot u = 0$$

Linearize governing equations

Linearization around temporal mean state, $\overline{\mathbf{q}}$, yields an equation describing the dynamics of the coherent fluctuation, $\widetilde{\mathbf{q}}$. In case of a turbulent flow, the interaction with the turbulent fluctuation, $\mathbf{q}'$, remains to be modeled.

$$[\mathbf{u}, p] = \mathbf{q} = \overline{\mathbf{q}} + \widetilde{\mathbf{q}} + \mathbf{q}' \Rightarrow \frac{d\widetilde{\mathbf{u}}}{dt} = \mathbf{L}\widetilde{\mathbf{q}}$$

Introduce harmonic modes

Modal ansatz (coherent fluctuation is harmonic in time) yields the linearized governing equations in frequency domain

$$\widetilde{q} = \widehat{q} e^{-i2\pi f t} \Rightarrow f\mathbf{B}\widehat{\mathbf{q}} = \mathbf{A}(\overline{\mathbf{q}})\widehat{\mathbf{q}}$$

Side note:
In the simplest case, the matrices A and B include one linearized transport equation for a passive scalar. In the most complicated case (reacting flows) they include the linearized transport equations of momentum, mass, enthalpy, six chemical species and take into account reaction rate. The vector q includes the respective state variables.

The numerical strategy of FELiCS

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Discretize linearized equations

All linearized flow equations can be expressed in differential form as $f\mathbf{B}\widehat{\mathbf{q}} = \mathbf{A}\left(\overline{\mathbf{q}}\right)\widehat{\mathbf{q}}$, which describes a set of coupled differential equations. Integrating the respective equations in the computational domain yields the respective discretized matrices.

Numerical implementation

- Implementation in Python: **Straightforward development** and **efficient** linear algebra interfaces with SLEPc and ARPACK
- Continuous Gallerkin Finite element package FEniCS: Well validated FEM framework
- Triangular/tetrahedral finite elements: **Complicated geometries are no challenge**
- Interfaces with commercial and non-commercial mesh generators
- Discretized matrices are sparse
- GUI allows for **straightforward application** to new flow configurations
- Running on Linux, Mac-OSX and Windows
- Discretization completely independent of type of analyis. Only the arrangement of matrices decides the what kind of analysis is performed (input-output analysis, resolvent analysis or linear stability analysis (see following slides))

Side note:
In most cases a symmetry of the coherent structure can be taken advantage of (e.g. homogeneous direction in Kelvin-Helmholtz roll-up, helicity of a precessing vortex core, rotation symmetry of a jet…) This allows addressing most problems in 2D, which enables us to solve all coupled equations simultaneously, by discretizing them in large, sparse matrices.

How to work with FELiCS

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Input to FELiCS

Temporal mean fields from:

- RANS
- LES
- DNS
- Experiments

Input

Computational grid, interface with:

- Gmsh
- Ansys ICEM
- Centaur

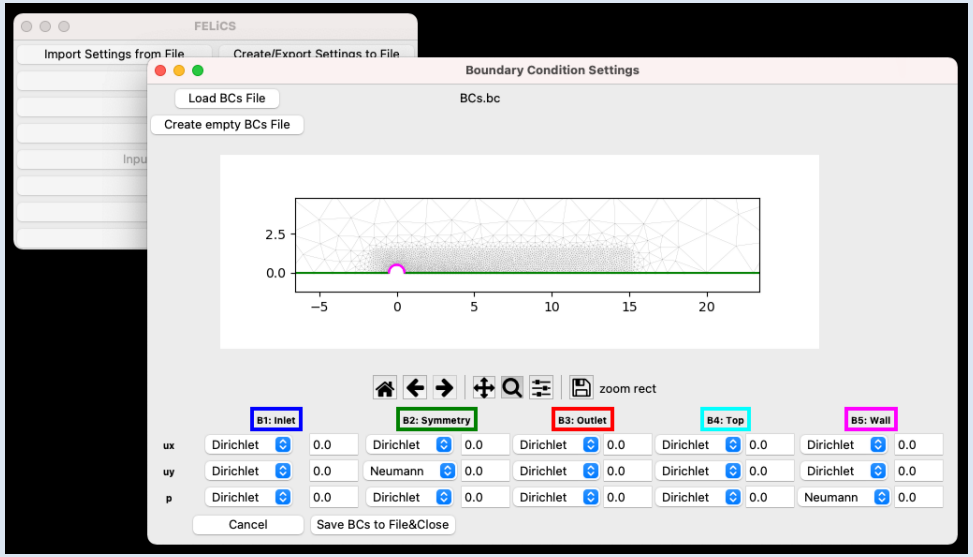
Input

FELiCS itself

Set case information in GUI or settings file. For example:

- Compressible or incompressible
- Reacting or non-reacting flow
- Boundary conditions and so on…

Example on using the GUI: Setting the boundary conditions for the cylinder wake flow (on a very coarse mesh)



Output of FELiCS:

The linear dynamics of the flow

Output

Linear stability analysis: Identify the intrinsic stability of the flow. Used for hydrodynamic instabilities.

Output

Input-output analysis: How does the flow respond to an incoming perturbation in the state variables (e.g. acoustic forcing)?

Output

Resolvent analysis: Identifies flow-intrinsic amplification mechanisms. Is widely used in turbulence research.

Example I: Input-output analysis of transport of passive scalar fluctuations in a turbulent channel flow

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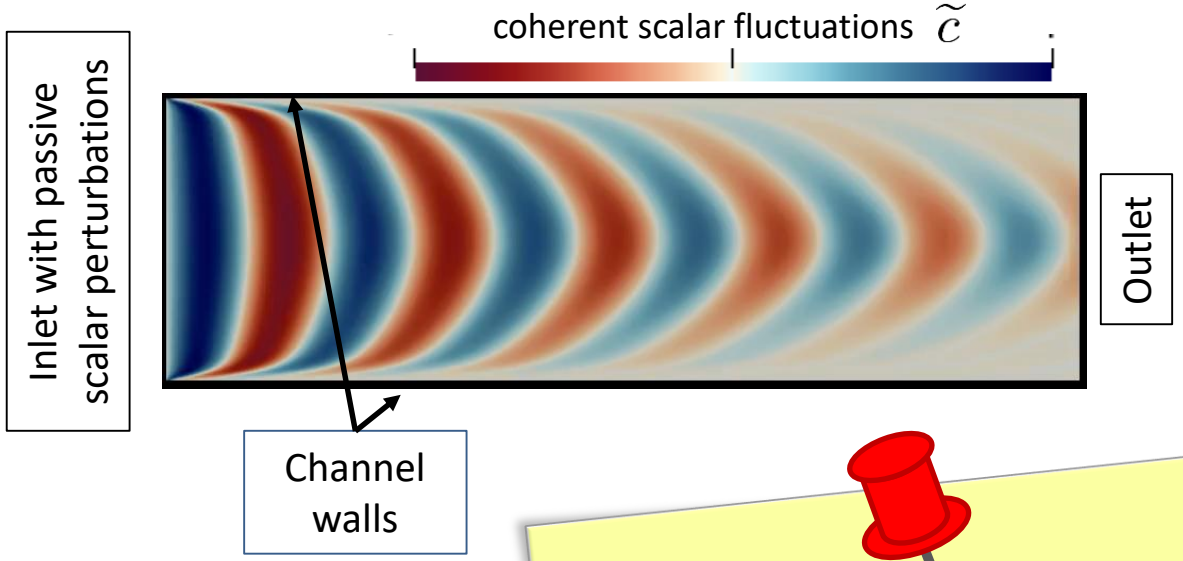
Turbulence modeling for closure

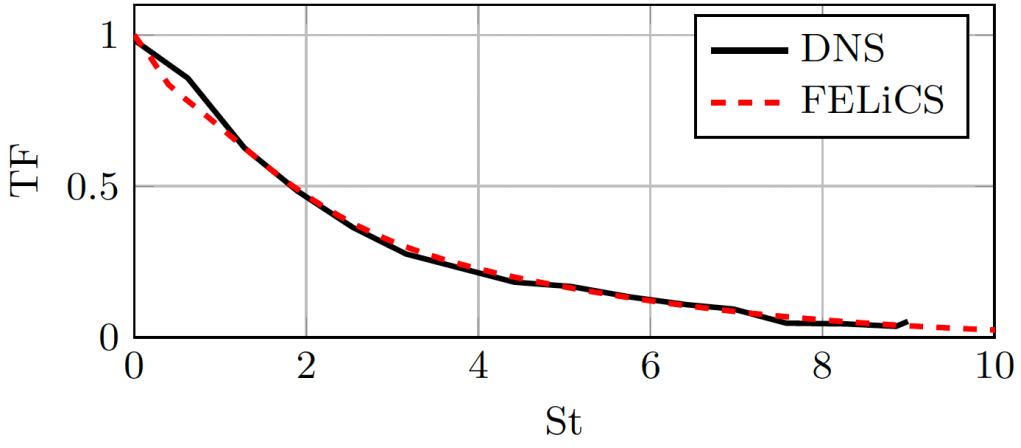
Non-vanishing product terms, $\widetilde{\mathbf{u}'c'}$, are modeled by an turbulent viscosity model, similar to RANS equations. The precision of this approach is tested in a turbulent channel flow and compared to DNS results.



Turbulent mixing (modeled by an eddy diffusivity in FELiCS) causes mitigation of passive scalar fluctuations as they are convected along the channel flow.

Comparison of DNS and FELiCS

The transfer function (TF) compares the amplitude of the passive scalar at the outlet with the inlet. FELiCS results obtained at negligible numerical cost (minutes on a single CPU for all frequencies) are in excellent agreement with DNS (Kaiser & Oberleithner, JFM 2021).



Example II: Resolvent analysis of a turbulent jet flame to identify the origin of Kelvin-Helmholtz vortex rings.

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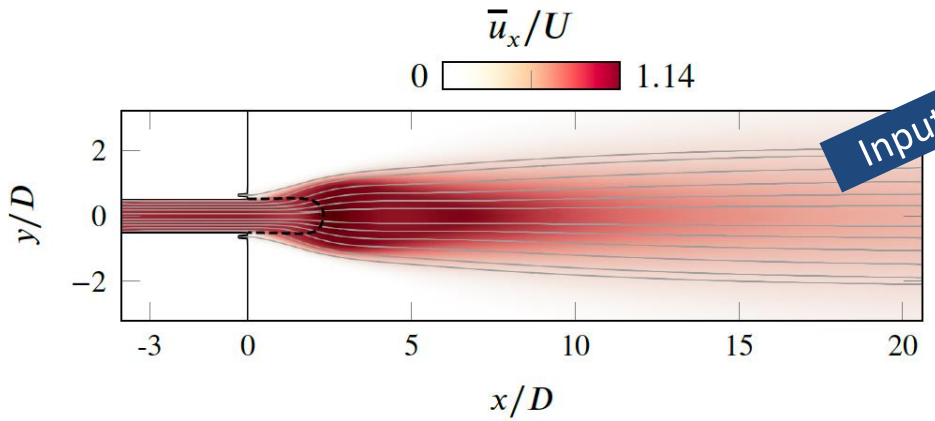
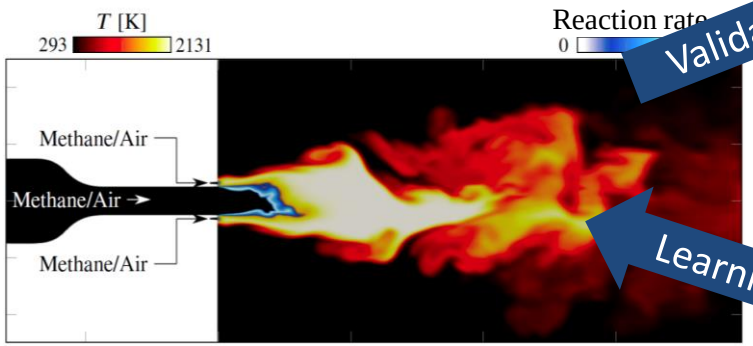
Temporal mean fields of flame (LES)



Input for

Instantaneous snapshot of flame dynamics (LES)



Validation for

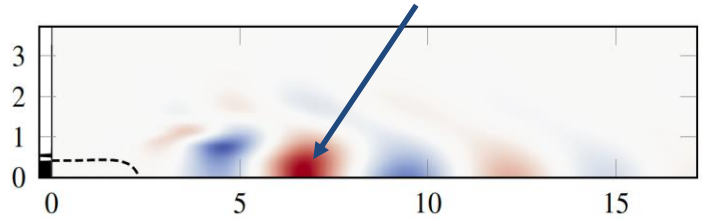
Learning about

Resolvent analysis with FELiCS

A resolvent analysis can be understood as an optimization problem. The goal is to find the spatial structure of a coherently oscillating forcing in the transport equations, $\widetilde{\mathbf{f}}$, which maximizes the objective function, $\mu = |\widetilde{\mathbf{q}}|/|\widetilde{\mathbf{f}}|$, where $\widetilde{\mathbf{q}}$ is the response to the periodic forcing.

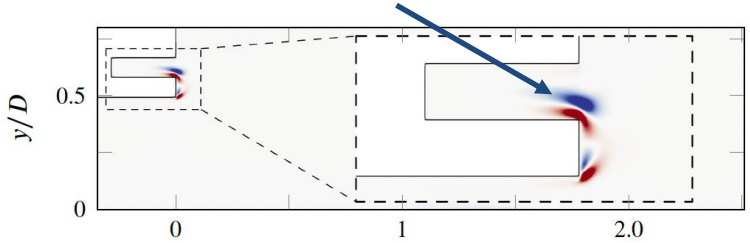
The optimal response obtained by the resolvent analysis with FELiCS identifies the dominant coherent structure in the flow: A Kelvin-Helmholtz vortex street. This is in agreement with the LES results.

Kelvin-Helmholtz vortices



The FELiCS resolvent analysis can do what an LES can not: Find the origin and, in doing so, the amplification mechanism that causes the dominant structure. The origin is located at the pilot burners.

Origin of the Kelvin-Helmholtz vortices



Example III: Linear stability analysis of the cylinder wake flow at the bifurcation point of the Karman vortex street (Re ≈ 47)

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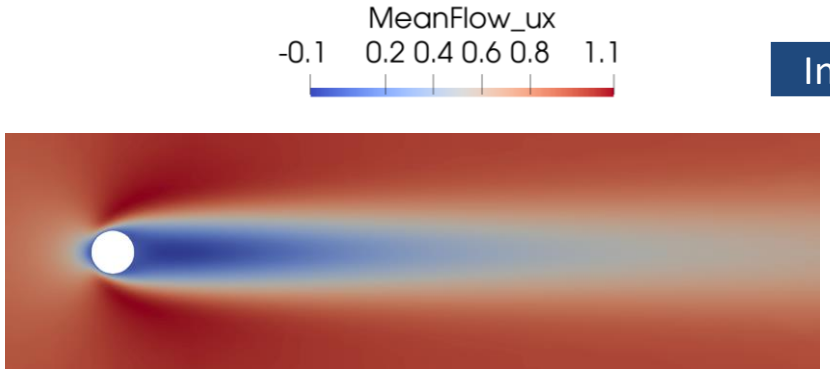
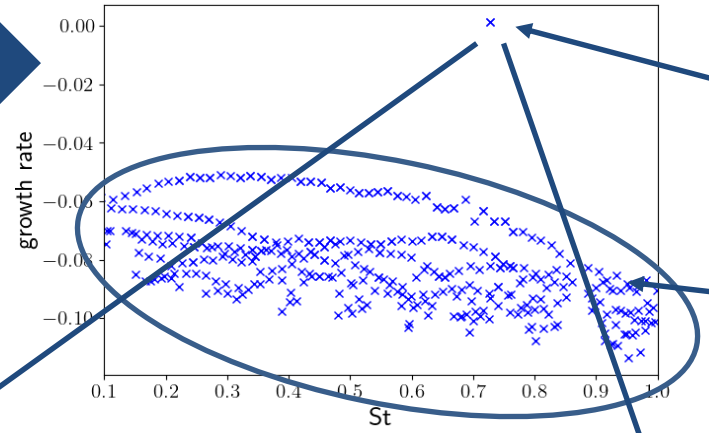
Temporal mean of cylinder wake flow



Input for

Linear stability analysis with FELiCS

Stability spectrum

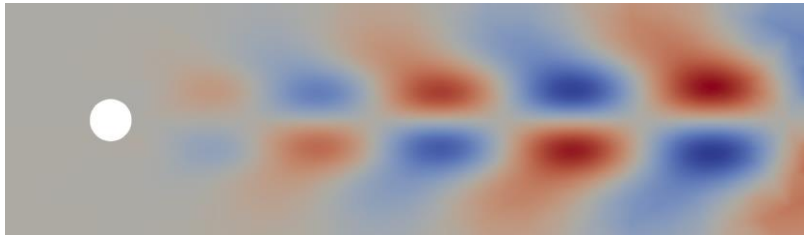


Single mode with zero-growth rate, indicating the bifurcation of the Karman vortex street

Various modes with negative growth rate

Mode shape of the Karman vortex street

Shows the spatial structure of the mode, the velocity elocity fluctuation caused by the Karman vortex street.



Adjoint mode shape

Shows the structure of a periodic forcing, which causes the strongest exicitation of the Karman vortex street.

Explains internal feedback, leading to the instability.

Current fields of applications

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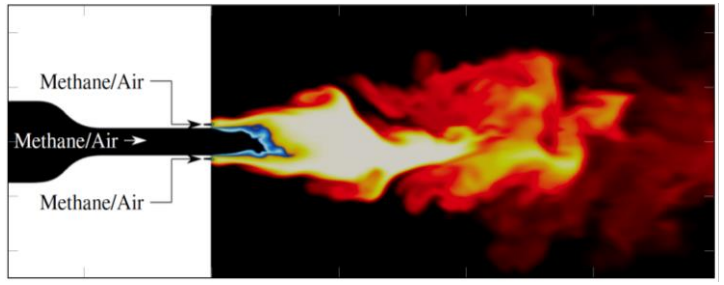
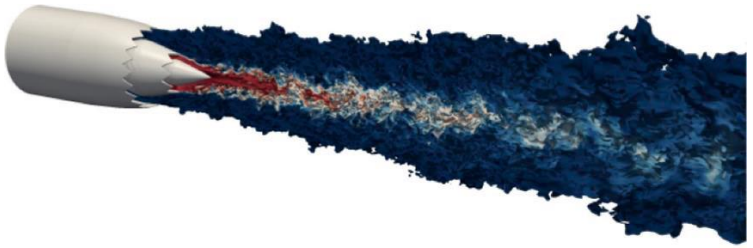
Flame turbulence (Casel et al. CnF 2021)

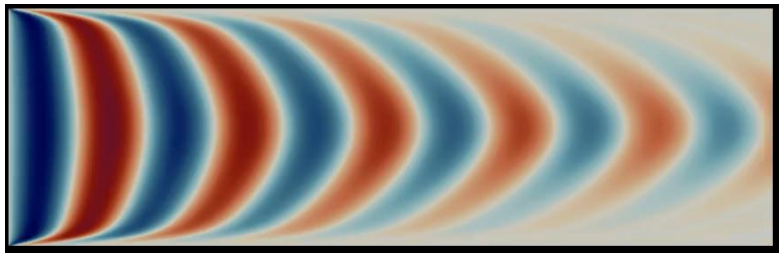


Jet turbulence (Kuhn et al. JFM 2021)

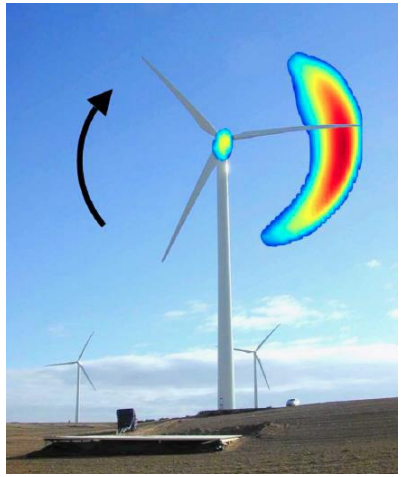


Advection/diffusion of entropy waves

(Kaiser et al. JFM 2021)

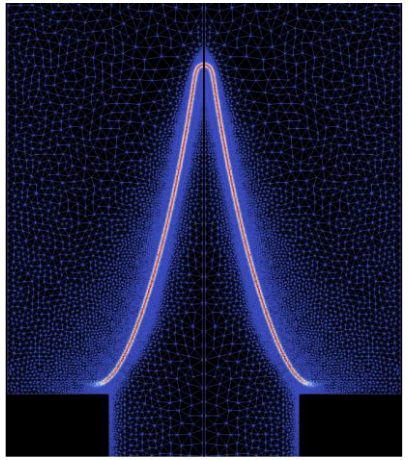


Airfoil noise



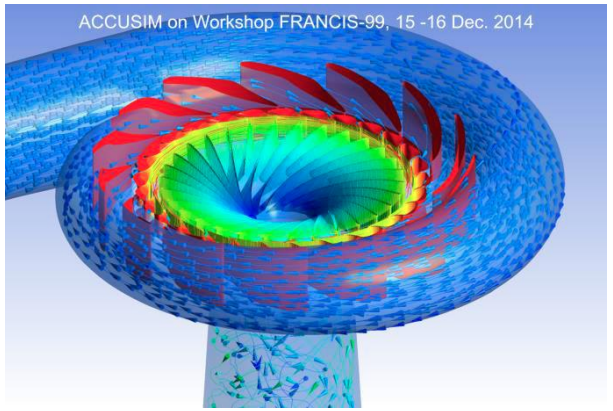
Linearized reacting flows

(Chuhan et al. CnF 2022)



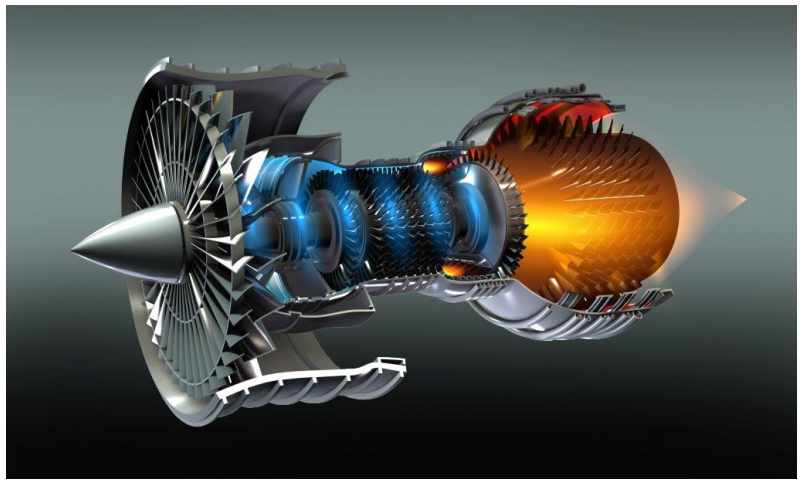
Instabilities in hydro turbines

(Müller et al. IAHR 2020)



Combustion instability

(Lückoff et al. CnF 2021; Müller et al. JGTP 2021)



Collaborations in the context of FELiCS

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Academia

Prof. W. Polifke @ Technical University Munich

- Thermoacoustics of turbulent flames

Prof. H. Bockhorn @ Karlsruhe Institute of Technology

- Combustion noise modeling

Dr. E. Pickering @ Massachusetts Institute of Technology (MIT)

- Turbulence closure modelling

Prof. L. Lesshafft @ Ecole Politechnique Paris

- Resolvent analysis

Prof. S. Alekseenko @ Institute of Thermophysics, Novosibirsk

- Hydro turbines

Prof A. Cavalieri @ Instituto Tecnológico de Aeronáutica, São José dos Campos, Brasil

- Airfoil noise

Industry

MAN Energy Solutions

- Thermoacoustic of stationary gas turbines
- Turbulent mixing and NOx reduction

Rolls-Royce Germany

- Thermoacoustics of jet engines
- Spectral analysis of LES

Voith (currently established)

- Hydro turbine instability
- Sensitivity analysis and flow control

References

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- Jens S. Müller, Finn Lückoff, Thomas L. Kaiser, C. Oliver Paschereit, Kilian Oberleithner: Modal Decomposition and Linear Modeling of Swirl Fluctuations in the Mixing Section of a Model Combustor Based on Particle Image Velocimetry Data. *Journal of Engineering for Gas Turbines and Power*, 144/1.

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- Finn Lückoff, Thomas Ludwig Kaiser, Christian Oliver Paschereit, Kilian Oberleithner: Mean field coupling mechanisms explaining the impact of the precessing vortex core on the flame transfer function. *Combustion and Flame*, 223.

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