Speculative Attacks with Multiple Sources of Public Information

Camille Cornand
BETA UMR 7522 CNRS
61, avenue de la Forêt noire
67085 Strasbourg Cedex France
cornand@cournot.u-strasbg.fr

Frank Heinemann
Technische Universität Berlin
Fachgebiet Makroökonomie, H 52
Straße des 17. Juni 135
10 623 Berlin, Germany
frank.heinemann@ww.tu-berlin.de

July 24, 2008[*]

Abstract:

We propose a speculative attack model in which agents receive multiple public signals. It is characterised by its focus on an informational structure, which sets free from the strict separation between public information and private information. Diverse pieces of public information can be taken into account differently by players and are likely to lead to different appreciations *ex-post*. This process defines players' expected private value of a successful attack. The main result shows that equilibrium uniqueness depends on two conditions: (i) signals are sufficiently dispersed (ii) private beliefs about the relative precision of these signals differ sufficiently. We derive some implications for information dissemination policy. Transparency in this context is multi-dimensional: it concerns the publicity of announcements, the number of signals disclosed as well as their precision. Especially, it seems that the central bank has better not publishing its forecast errors in order to maintain stability. An illustration to our analysis is the recent debate concerning the optimal monetary policy committee structure of central banks.

Keywords:

Speculative attack – coordination game – multiple equilibria – public and private information – transparency.

JEL Classification:

F31 – D82.

---

[*] We are grateful to Jean-Pierre Allegret, Agnès Bénassy-Quéré, André Cartapanis, Hendrik Hakenes, Philippe Martin, Timofiy Mylovanov, Hyun Shin as well as two anonymous referees and Espen R. Moen, the editor of the *Scandinavian Journal of Economics*, for helpful comments and to Alexander Meyer-Gohde for helping us with simulations. Camille Cornand acknowledges financial support from the U.K. ESRC under grant RES-156-25-0026 ("Stability of the Global Financial System: Regulation and Policy Response", World Economy and Finance, ESRC Research Programme Phase I) and thanks the FMG for hospitality. Finally, both authors acknowledge financial support from the joint grant ANR-DFG.

1 – Introduction

Speculative attacks are based on information which is in parts publicly available or provided by media and agencies that are recognised by all major traders on foreign exchange markets. Public information is not just helpful in predicting the future course of an economy, but also induces higher order beliefs that allow for crises occurring out of self-fulfilling beliefs. In this paper, we analyze whether multiple sources of public information prevent self-fulfilling prophecies.

Second generation speculative attack models in the tradition of Obstfeld (1986, 1996) can be modelled as coordination games with multiple equilibria. Whether a central bank devaluates a currency depends on market pressures that arise from traders' beliefs about the probability of devaluation. If traders believe in devaluation and speculate against a currency, market pressure may force a central bank to abandon a peg that it would have kept without the additional pressure generated by speculators. Applying the global-game approach, Morris and Shin (1998) have shown that this kind of coordination games has a unique equilibrium, if traders' information is private instead of public.[1] Morris and Shin (2003) and Hellwig (2002) show that equilibrium uniqueness requires that agents attach a sufficiently large weight to private information when both, private and public signals are available. Bayesian rationality requires that weights are positively related to the precision of information, which is the inverse of the variance of the respective signals. Thus, uniqueness relies on private signals being sufficiently precise compared to public signals. In the real world, however, the most precise information is provided publicly by transparent central banks and well-informed agencies. This raises concerns about whether economic transparency may lead the inclination to self-fulfilling prophecies. A counterargument to this view is provided by Lindner (2006) who defines transparency as providing detailed public information. The details may be viewed as different signals, each of which can only be used for Bayesian updating in combination with some agents' private information. Thereby, public signals are actually improving the precision of traders' private information. Greater precision of private information, however, stabilizes the economy by preventing multiple equilibria. Hence public announcements may actually contribute to stabilizing an economy.

In this paper, we develop another argument for equilibrium uniqueness: even if central banks' announcements are publicly observable and may be thought of being common knowledge, their precision is not. In the presence of multiple signals, Bayesian updating requires to attach some weight to each signal that is related to the signals' relative precision. If agents have different beliefs about the precision of signals, they arrive at different posterior expectations. In the paper we show that sufficient dispersion of beliefs about signals' precision leads to a unique continuation equilibrium. In other words: with sufficient dispersion of beliefs, the actions that are chosen by traders are uniquely determined. Whether or not this condition holds depends on the number and precision of public signals, on available information about their relative precision, and on the actual realization of signals. We derive some conclusions for the optimal dissemination of information and show that a central bank can reduce the probability of multiple equilibria by providing multiple imprecise signals and withholding information about the relative precision of these signals.

Our starting point is that different agents treat the same information differently and posterior beliefs are private information even though agents have a common prior and all information about economic fundamentals is publicly available. Agents arrive at different posterior beliefs, because they receive several signals of unknown precision and hold private beliefs about these distribution parameters.[2] While many figures about an economy are provided publicly and become common knowledge (at least in theory), the precision of these figures is usually not public information. A rare exception is the report by the Bank of England that publishes "fan charts" in addition to inflation forecasts.[3] Also committees rather than individuals more and more take decisions in central banks and although decisions are very often consensual, committee members might express their own view either informally or by voting. This is especially the case for the Bank of England (with the publication of *the minutes* of monetary policy committee discussions). With multiple public signals, beliefs about the relative precision of these signals may differ between agents and lead to different posterior beliefs about the state of the world.

We introduce multiple sources of public information in the currency-attack model by Morris and Shin (1998, 2003). Agents receive noisy public signals and have different opinions about the relative precision of these signals. We analyze conditions for uniqueness of the equilibrium. The model has a unique equilibrium, if there are sufficiently many or strong announcements that hint at states at which either attacking or not-attacking are dominant strategies. In addition, there must be a sufficient mass of agents who attribute enough weight to these signals, so that attacking or not-attacking, respectively, are dominant strategies given their posterior beliefs.

We restrict our formal analysis to three cases distinguished by the number of public signals. Each of the three cases gives an additional insight for the intuition that carries over to a more general framework. If there are just two signals and agents' beliefs about the relative precision of these signals have a uniform distribution, there is a unique equilibrium if and only if at least one of the signals indicates a state at which either attacking nor not-attacking is a dominant strategy. With more than two signals or with a uni-modal distribution of beliefs about their precision, uniqueness may require that signals are sufficiently different and agents put a sufficiently strong weight on the most extreme signals. When the number of signals approaches infinity, the distribution of posterior beliefs becomes

---

[1] A signal is private information if it is received by a single agent and public information if it is received by all agents, all agents know that all agents received the same signal, and so on.

[2] In this sense, they have non-common priors about distribution parameters.

[3] Fan charts indicate estimated probabilities for future inflation rates. These probabilities account for estimated forecast errors, but not for possible errors in the model underlying these estimates.

common knowledge. This turns the private-information game into a private-value game, for which we know that it has a unique equilibrium, provided that there is a sufficient mass of agents for whom either action is a dominant strategy (Dönges and Heinemann, 2006). In our model, this requires that the average precision of public signals is sufficiently low.

In terms of economic policy, we conclude that the central bank should benefit from at least two tools: if used appropriately, number and precision of public announcements can be effective at stabilising the economy in situations where it might be prone to self-fulfilling crises otherwise. The provision of different specialized data about the fundamentals of an economy reduces the inclination to self-fulfilling prophecies in comparison to the provision of just one compound announcement. With a sufficiently large number of public signals, the probability that an economy is hit by a crisis due to self-fulfilling beliefs can be reduced to almost zero, provided that these signals are not too precise.

Section 2 introduces the model. In Section 3 we show how uniqueness of the equilibrium is related to the number and distribution of signals and to the distribution of private beliefs about their relative precision. Section 4 reports results from a simulation study: here we solve the model numerically for different parameter values and demonstrate that a central bank can stabilize the economy by disclosing information in small pieces instead of publishing summary statistics. In section 5 we draw some lessons for the optimal modes of information dissemination and discuss how these considerations can be understood for the conduct of monetary policy, especially *via* committee structure. Section 6 concludes the paper by summing up main results.

2 – Model

The model builds on the reduced form of a currency-attack game introduced by Morris and Shin (1998, 2003). It deals with an open economy in which the central bank has anchored its exchange rate on a fixed parity. Our main innovation is the introduction of multiple public signals of unknown precision.

2.1. Reduced form game

There is a continuum of risk neutral agents (speculators) $i \in [0,1]$ who decide simultaneously whether or not to attack a currency peg by short selling one unit of domestic currency. An attack is associated with transaction costs $t > 0$ that are linked to the differential of interest rates between domestic and foreign currency. The fundamentals of the economy are summarized by an aggregate state variable θ. If the proportion of agents who decide for attacking the currency exceeds θ, the central bank devaluates the currency and attacking agents earn an amount $R > t$. If the proportion of attacking agents is less than or equal to θ, the central bank keeps the peg and attacking agents just loose transaction costs. A high (low) value of θ represents a good (respectively bad) fundamental state. If

$\theta \geq 1$, the economy is in a sound condition where the central bank can always defend the currency against an attack. If $\theta < 0$, the currency must be devaluated even without the additional market pressure from speculating traders. The aggregate state θ may be interpreted as a measure of the additional market pressure from speculation that is needed to enforce devaluation. We assume that θ has a normal distribution with mean θ_0 and variance σ^2.

If a speculator knows that $\theta < 0$, attacking is a dominant strategy, because it leads to a positive payoff independent of the other traders' actions. If a trader knows that $\theta \geq 1$, it is a dominant strategy not to attack, because an attack cannot be successful. If it is common knowledge amongst traders that $0 \leq \theta < 1$ there are two equilibria in pure strategies: either all traders believe in devaluation and attack the currency. In this case, the central bank gives in and beliefs turn out to be correct. Or, traders do not believe in devaluation and abstain from an attack. In this case, the central bank keeps the peg and beliefs are also fulfilled. In addition, there are mixed-strategy equilibria in which the probability that the proportion of attacking traders exceeds θ equals t / R.

2.2. Different informational assumptions

Morris and Shin (2003) distinguish private and public information by assuming that each agent *i* receives two signals, x^i and *y*, that differ from θ by independent noise terms with normal distributions. The variances of $x^i - \theta$ are the same for all *i*. Signal x^i is private information of agent *i*, while the public signal *y* is commonly observed by all agents.[4]

If the variance of private signals is sufficiently small compared to the variance of the public signal, so that

$$Var(y - \theta) > \sqrt{\frac{Var(x^i - \theta)}{2\pi}},$$

the game has a unique Bayesian equilibrium with a threshold function $x^*(y)$, such that for a given public signal *y* all agents with signals $x^i < x^*(y)$ attack the currency, while agents with higher signals do not attack. The intuition is that when the precision of these private signals is sufficiently high (compared to that of the public signal), agents will rather rely on their private information and the probabilistic environment (generated by the existence of private signals) adds an equilibrium condition that is sufficiently strong to characterize a unique equilibrium (as in Morris and Shin (1998)). The higher order beliefs generated by the presence of private signals enable Morris and Shin to apply what

[4] Morris and Shin assume that the prior distribution of θ is common knowledge, so that $y = E(\theta)$. But, the results are not affected by assuming that the prior mean is unknown and agents receive some other public signal instead.

Myatt, Shin and Wallace (2002) call the infection argument.[5] Even if all agents have private information telling them that the fundamental is in the intermediate region, this event is not common knowledge, which prevents self-fulfilling beliefs about success of an attack. Otherwise (if the precision of private information is low compared to that of public information), the game has multiple equilibria, so that for some realizations of *y*, the threshold to attack is not uniquely determined. Indeed, when public information is rather precise, agents ignore their private signal and coordinate on the public one, which leads to a situation of common knowledge generating multiple equilibria as in the standard model of Obstfeld (1996). Here, an attack can be triggered by events that are unrelated to economic fundamentals (sunspots), because traders' beliefs are self-fulfilling. In this light, transparency can have destabilizing effects: if central banks provide accurate information about their foreign currency reserves and publish their statistics and predictions about the future course of the economy, the high precision of this information raises the danger of sudden currency crises triggered by unpredicted shifts of beliefs.

Metz (2002) and Bannier and Heinemann (2005) analyze the comparative statics of the equilibrium with respect to signals' precisions, provided that the condition for uniqueness holds. Heinemann and Illing (2002) suggest that public information should be intermediated by private agencies to prevent agents from exactly inferring which information other agents possess. The idea is that each agent gets information on demand without knowing whether other agents possess the same information as to avoid common knowledge. All of these papers assume that there is a single public signal.

While endowing agents with heterogeneous information exogenously guarantees a unique equilibrium in a variety of strategic settings, Hellwig and Veldkamp (2007) show that heterogeneous beliefs may be difficult to sustain if agents can choose information endogenously. With strategic complementarities, the game of information acquisition imposes an additional requirement for equilibrium uniqueness: the information that agents choose to acquire must be private (and signal noise across agents must be independent). The main idea in Hellwig and Veldkamp (2007) is that the information economic agents observe not only depends on its availability but also on their choice of what to learn. Comparing games with and without information acquisition, they highlight that when agents choose how much information to acquire before choosing strategic actions, the information choice inherits strategic complementarity from the actions. In their terms: "*Agents who want to do what others do want to know what others know*" (p. 2). This complementarity in information acquisition can cause multiple equilibria to resurrect, even if the exogenous-information model has a

[5] Intuitively, the idea is the following. A player who believes that $\theta \geq 1$ does not attack. Hence a player who believes that $\theta = .99$ attributes some probability to her opponent believing that $\theta \geq 1$ and hence will also abstain from an attack. Now, a player who believes that $\theta = .98$ suspects that her opponent might believe that $\theta \geq .99$ and hence will not attack either, and so on. Similarly, we can consider a range of values for which a player finds it optimal to attack, working upwards from $\theta = 0$. The argument amounts to an iterative elimination of

unique equilibrium. Agents want to invest in additional public information if and only if others do so as well, and so the equilibrium level of public information is indeterminate. However, if agents invest only in private signals, complementarities are weak resulting in a unique equilibrium.

Another line of critique to the global-game approach arises from its assumption that public signals are independent from agents' actions. Angeletos and Werning (2006) as well as Ozdenoren and Yuan (2008) consider coordination games, in which the publicly observable price of an asset partially reveals private information and, thereby, gives rise to multiple equilibria. Similarly, Hellwig, Mukherji, and Tsyvinski (2006) consider speculative-attack games, in which the interest rate aggregates all private signals about central bank reserves and, thereby, publicly reveals these reserves. This also results in multiple equilibria. The interest rate, however, is an instrument of the central bank. By adjusting this rate, the central bank disseminates information. Central banks are well aware of the fact that their decisions provide market signals and deliberately use their instruments to simultaneously affect market conditions and beliefs. We do not mix these two properties in our paper, but rather concentrate on the revelation of information.

The approach of deriving uniqueness by assuming that public information is less precise than available private information has been criticized, because in reality the most relevant information is publicly available. Romer and Romer (2000) provide evidence that economic forecasts, published by the Federal Reserve System are more accurate than private forecasts. The means of disclosing information by central banks are designed to achieve the highest possible publicity. Even if these means are changed, strategic complementarity provides incentives for private agents to share their information with others, so that all information may be publicly available.

On the other hand, public information is not homogeneous: there is a plurality of channels *via* which information is provided to the public and even central banks publish different kinds of information that may be more or less relevant for predicting future exchange rates. Central bank committee members express their own views that are sometimes conflicting with the decisions taken by the committee as a whole. Private agents may spread their information to induce other traders taking the same position, but they also have incentives to misrepresent their own information, which limits credibility of public signals from private sources. Available public signals differ in their relevance for predicting the aggregate state θ and in their precision. While public signals themselves may be thought of being common knowledge, relevance and precision of these signals are in general not commonly known. Hence, traders may disagree on the relative importance of these signals for predicting θ. In the absence of a commonly agreed model (or a common prior) agents may even be aware about their different evaluations and agree to disagree. Consequently, agents may hold different posterior beliefs even if they all receive the same signals. This raises the question whether multiple

dominated strategies and may (depending on distribution parameters) lead to a unique threshold θ^*, such that a player attacks if and only if $\theta < \theta^*$.

sources of public information with unknown precisions are sufficient to guarantee a unique equilibrium.

2.3. Multiple public signals

We extend the currency-attack model of Morris and Shin (1998, 2003) by introducing $K > 1$ public signals received by speculators. Signals are given exogenously and can be observed free of costs by all agents. Each signal y_k differs from the fundamental state θ by a noise term with a normal distribution, *i.e.* $y_k = \theta + \varepsilon_k$, with $\varepsilon_k \sim N(0, \tau_k^2)$. Noise terms ε_j and ε_k are independent for all $j \neq k$. Denote the vector of public signals by $Y = (y_1, y_2, ..., y_K)$. We interpret each k as one source of public information. To simplify the formal exposition, we assume that the prior mean θ_0 is not observed.[6] Since agents do not know the prior, they are assumed to start out with an improper (but common) prior as in Morris and Shin (2003, Section 2), that is a uniform distribution on the reals. Each agent takes into account the whole vector of K commonly observable signals. But, agents do not know the true variances and attribute subjective weights to each of these signals. Thus, they have different (prior) beliefs about signals' variances τ_k^2.

The posterior expectation associated with a vector of normally distributed signals Y is a weighted average of these signals, $E(\theta \mid Y) = \sum_{k=1}^{K} q_k y_k$, where the weights are given by the relative precision (inverse variance) of these signals

$$q_k = \frac{\frac{1}{\tau_k^2}}{\sum_{k=1}^{K} \frac{1}{\tau_k^2}}.$$

The posterior variance is given by

$$V(\theta|Y) = \frac{1}{\sum_{k=1}^{K} \frac{1}{\tau_k^2}}.$$

If the variances of public signals were known, all agents would agree in their posterior expectation, and the model would be indistinguishable from a model with a single public signal. To avoid this, we assume that agents do not know the true variances and have private beliefs about the signals' precisions, instead. To keep the model tractable, we assume that agents agree on the aggregate level of

[6] One might view the model as a snapshot of time, so that the prior mean might become common knowledge. Considering this does not affect the results, because the prior mean can be modelled as being one out of many public signals, say $y_1 = \theta_0$ and therefore $\tau_1 = \sigma$, as long as agents disagree about relative precisions. Note, however, that numerical simulation results in Section 4 are affected by this.

uncertainty in the economy. In particular, we assume that $V(\theta|Y)$ is common knowledge. Thus, agents differ only in their beliefs about the *relative* precisions of public signals that determine the weights q_k attributed to signals in the posterior expectation of the fundamental state.

We denote the weights that agent *i* attaches to public signals by $q^i = (q_1^i, \ldots, q_K^i)$. Of course, these weights must sum up to one, so that they are contained in a *K*-dimensional simplex,

$$q^i \in \Delta^K = \left\{ q \in \Re^K \;\middle|\; 0 \le q_k \le 1, \quad \sum_k q_k = 1 \right\}.$$

An agent who believes that relative precisions are given by q^i has a posterior subjective belief about θ that is described by a normal distribution with $E^i(\theta|Y) = \sum_k q_k^i y_k$ and $V^i(\theta|Y) = V(\theta|Y)$.

2.4. Dominant strategies

Because of strategic complementarities, the expected payoff from an attack by some agent *i* rises in the proportion of other agents who are attacking. Suppose no other agent attacks. Then, the currency will be devaluated if and only if $\theta < 0$. The payoff that agent *i* expects from attacking is

$$R \Pr\left(\theta < 0 \middle| Y, q^i\right) - t = R\,\Phi\left(\frac{0 - E^i(\theta|Y)}{\sqrt{V(\theta|Y)}}\right) - t,$$

where $\Pr(\theta < 0 | Y, q^i)$ denotes the subjective probability that an agent with beliefs q^i attributes to the event $\theta < 0$. Given the normality of subjective conditional distributions, we can express the expected payoff using the cumulative standard normal distribution Φ. Thus, agent *i* has an incentive to attack, even if no other agent attacks, provided that

$$R\Phi\left(\frac{0 - E^i(\theta|Y)}{\sqrt{V(\theta|Y)}}\right) > t \quad \Leftrightarrow \quad E^i(\theta|Y) < -\sqrt{V(\theta|Y)}\;\Phi^{-1}(t/R) = \underline{\theta}.$$

In other words, it is a dominant strategy for an agent to attack if her subjective posterior belief is below $\underline{\theta}$.

Suppose now that all agents attack. Then, the currency will be devaluated if and only if $\theta < 1$. Now, agent *i*'s expected payoff from attacking is positive if

$$R\Phi\left(\frac{1 - E^i(\theta|Y)}{\sqrt{V(\theta|Y)}}\right) > t \quad \Leftrightarrow \quad E^i(\theta|Y) < 1 + \underline{\theta} = \overline{\theta}.$$

In other words, it is a dominant strategy for an agent to abstain from an attack if her subjective posterior belief is above $\overline{\theta}$.

In the benchmark case with just one public signal (*K*=1), there are multiple continuation equilibria if and only if the public signal is contained in $[\underline{\theta},\overline{\theta})$.

2.5. Bayesian equilibria

Players are distinguished only by their weights on public signals. The vector of weights q^i defines a player's type, and Δ^K is the type space. Denote the distribution of types by *g* and assume that *g* is continuous on Δ^K. As usual in a Bayesian equilibrium, we assume that the distribution of types is common knowledge.[7]

A strategy is a function $a:\Delta^K \times \Re^K \to [0,1]$, where $a(q^i,Y)=a^i(Y)$ denotes the probability that agent *i* attacks if she observes the vector of public signals *Y*. For a given vector of public signals *Y*, the proportion of attacking speculators is

$$\int_0^1 a^i(Y)\,di = \int_{\Delta^K} a(q,Y)\,dg \quad .$$

The central bank devaluates the currency if this proportion exceeds θ. Thus, for any vector of public announcements *Y* and for any strategy *a*, the currency will be devaluated if and only if

$$\theta < \theta^*(Y) = \int_0^1 a^i(Y)di \,.$$

Thereby, the decision problem of a single agent boils down to attack if and only if the subjective probability for the state being worse than some threshold $\theta^*(Y)$ is sufficiently large.

The expected payoff from an attack for agent *i*, given the vector of public signals *Y*, and the agent's subjective beliefs q^i, is

$$R\ \Pr\left(\theta < \theta^*(Y)\middle| Y, q^i\right) - t \;=\; R\Phi\left(\frac{\theta^*(Y)-E^i(\theta|Y)}{\sqrt{V^i(\theta|Y)}}\right) - t \,.$$

Agent *i* attacks the currency if the expected payoff is positive, which is equivalent to

---

[7] In this model, agents agree to disagree. With common priors about the signals' relative precisions, rational agents cannot agree to disagree (Aumann, 1976). Thus, our model requires heterogeneous priors. We do not see this as a problem, because there is no reason why prior beliefs about $\{\tau_k\}$ should be identical and agents cannot directly observe others' beliefs. They can at best deduce the distribution of private beliefs from some market prices that we did not explicitly consider, because we assume that the distribution of private beliefs is common knowledge anyway.

$$E^i\left(\theta \middle| Y\right) < \theta^*\left(Y\right) - \sqrt{V\left(\theta \middle| Y\right)}\,\Phi^{-1}\left(\frac{t}{R}\right) = \theta^*\left(Y\right) + \underline{\theta}. \tag{1}$$

Recall that conditional variances are the same for all agents. Equation (1) shows that an agent attacks if her posterior expectation is below some threshold, at which the expected reward from an attack equals its costs. The proportion of attackers is the proportion of all agents with subjective expectations below this threshold. In equilibrium, the marginal state $\theta^*(Y)$, below which the central bank abandons the currency peg, is given by the proportion of agents who attack if the currency is devaluated for all $\theta < \theta^*(Y)$. This gives the equilibrium condition

$$\theta^*(Y) = \left| \left\{ i \in [0,1] \middle| E^i(\theta \mid Y) < \theta^*(Y) + \underline{\theta} \right\} \right|, \tag{2}$$

where $\left|\{\Lambda\}\right|$ is the Euclidian size of the subset. Note that for a given vector of public signals *Y*, distribution *g* implies a distribution of posterior beliefs $E^i(\theta \mid Y)$. Denote this distribution by f_Y and the associated cumulative distribution by F_Y. Formally, this distribution is defined by

$$F_Y(\theta) = \Pr\left(\sum_{k=1}^{K} q_k^i\, y_k < \theta \middle| Y, g \right) = \left| \left\{ i \in [0,1] \middle| \sum_{k=1}^{K} q_k^i\, y_k < \theta \right\} \right|.$$

The equilibrium condition can now be rewritten as

$$\theta^*(Y) = F_Y\left(\theta^*(Y) + \underline{\theta}\right). \tag{3}$$

The associated equilibrium strategy is $a^{*i}(Y) = 1$ if $E^i\left(\theta \middle| Y\right) < \theta^*\left(Y\right) + \underline{\theta}$ and $a^{*i}(Y) = 0$ if $E^i\left(\theta \middle| Y\right) > \theta^*\left(Y\right) + \underline{\theta}$. For $E^i\left(\theta \middle| Y\right) = \theta^*\left(Y\right) + \underline{\theta}$, the equilibrium strategy associated with θ^* is not uniquely defined. Since *g* is continuous, these players have mass zero, so that they do not affect the aggregate outcome.

The described game has two stages: first nature selects realizations of random variables *θ* and *Y*. Then, players decide on whether or not to attack. It is straightforward to see that equilibrium strategies are not unique, because there exist multiple continuation equilibria in certain subgames, *i.e.* for certain realizations of *Y*. Consider, for example, the case where all signals hint at some intermediate state of the economy, that is $\underline{\theta} < y_k < \overline{\theta}$ for all *k*. All agents' posterior beliefs are a weighted average of these signals, so that $\underline{\theta} < E^i(\theta|Y) < \overline{\theta}$ for all *i*. For these posteriors, an attack has a positive expected payoff if all agents attack and a negative expected payoff if almost nobody attacks. Agents agree to disagree in their posterior expectations, but it is common knowledge that everybody believes an attack to be rewarding if everybody attacks, and to fail if almost nobody attacks. The subgame has multiple equilibria as in the benchmark case with just one public signal. There are at least three solutions to equation (3), $\theta^*(Y) = 0$, $\theta^*(Y) = 1$, and at least one equilibrium with $0 < \theta^*(Y) < 1$ in which agents with expectations below some interior threshold attack.

We are interested in the conditions on *Y* and *g*, for which subgames starting with *Y* have a unique equilibrium. Given that *Y* is a random variable, we can then ask how likely it is that there is a unique continuation equilibrium.

It is a necessary condition for a unique continuation equilibrium that at least one of the signals is outside the intermediate region $(\underline{\theta}, \overline{\theta})$. Whether the equilibrium is unique or not depends on the vector of public announcements *Y* and on the distribution of private beliefs q^i. An explicit solution of uniqueness conditions requires special assumptions for the number of signals and for the distribution of subjective weights. To get an intuition for general uniqueness conditions, we characterise them for particular distributions of private weights q^i and for three special cases for the number of signals, $K = 2$, $K = 3$ and $K \to \infty$. Then, we explain the rationale that carries over to general settings.

3 – Equilibrium uniqueness

In this section, we derive conditions for uniqueness of continuation equilibria. We show that multiple public signals can lead to a unique equilibrium, even if the objective posterior hints at a state at which an attack may occur out of self-fulfilling prophecies.

We start our analysis with the case of $K = 2$ signals and a uniform distribution of beliefs q^i. In this case, a unique equilibrium exists if and only if one signal falls outside the intermediate region. While the two-dimensional case is useful for illustrating the consequences of private information about variances, the result is not robust with respect to the number of signals or with respect to the distribution of beliefs. By contrast, the game with $K = 3$ public signals has a unique equilibrium if there is sufficient dispersion between the highest signal and the lowest signal. This case yields robust insights in the interaction between the particular signals and the distribution of private beliefs for the determinacy of equilibrium behaviour: uniqueness of a continuation equilibrium requires a sufficient dispersion of public signals. This dispersion must be wider the more concentrated the distribution of private weights is. Finally, we solve the case for an infinite number of public signals. Our analysis shows how the accuracy of public announcements affects the existence of multiple equilibria: uniqueness requires the precision of public signals to be sufficiently low.

For our formal analysis we assume that subjective weights q^i have a uniform distribution on the *K*-dimensional unit simplex. The corresponding density function is $g(q^i) = 1/S$, $\forall q^i \in \Delta^K$, where

$S = |\Delta^K|$ is the Euclidian size of the *K*-dimensional unit simplex. Without loss of generality, we assume that $y_1 \le y_2 \le \Lambda \le y_K$.

3.1. Equilibrium in the case of two public signals

Suppose there are just two public announcements, y_1 and y_2. We know already that there are multiple equilibria, if both signals are in the interval $(\underline{\theta}, \overline{\theta})$. Now assume, instead, that one signal hints at a particular bad state at which an attack is a dominant strategy, *e.g.* $y_1 < \underline{\theta}$. Then, there is a positive mass of agents, who believe that attacking is a dominant strategy. Since the distribution of posteriors F_Y is common knowledge, other agents know that there are some agents, for whom attacking is a dominant strategy. Thus, they expect a critical mass of attacking capital that raises their own threshold up to which an attack appears promising. Agents with higher posteriors attack, because they can deduce that at least a certain fraction of agents attacks. Since other agents know this as well, some traders with even higher posteriors attack, and so on.

With one signal in the “attack” region, $y_1 < \underline{\theta}$, and the other in the multiplicity region, $\underline{\theta} < y_2 < \overline{\theta}$, there exists one equilibrium, in which all agents attack. Here, the elimination procedure may eliminate any other equilibrium. Vice versa, if there is one signal in the “not attack” region, $y_2 > \overline{\theta}$, and the other is in the multiplicity area: there exists one equilibrium, in which no agent attacks and it may be the only one. Whether the elimination process stops before the threshold reaches the other signal and there are multiple equilibria or not, depends on the distribution of private weights q^i.

If there is one signal in each of the two extreme regions, the elimination procedure reduces the multiplicity region from both sides and may lead to a unique equilibrium with an intermediate threshold, such that all agents with pessimistic beliefs (below the threshold) attack, while agents with more optimistic beliefs refrain from attacking. Whether the elimination from both sides stops at the same point and yields a unique equilibrium or not, depends once more on the distribution of private weights. Uniqueness requires that at least one signal is outside the multiplicity region and that the distribution of beliefs puts sufficient weight on the worst and/or the best realized signals, so that enough mass is attracted in each step of the elimination procedure. For a uniform distribution of weights q^i on the simplex Δ^2, we can show that there are multiple equilibria if and only if *both* signals are inside the multiplicity region.

Suppose (without loss of generality) that $y_1 < y_2$. Now, the equilibrium condition (3) can be expressed as

$$\theta^* = \left| \left\{ q \in [0,1] \middle| \, q y_2 + (1-q) y_1 < \theta^* + \underline{\theta} \right\} \right| = \left| \left\{ q \in [0,1] \middle| \, q < \frac{\theta^* + \underline{\theta} - y_1}{y_2 - y_1} \right\} \right|.$$

An equilibrium with $\theta^* = 0$ exists, whenever $y_1 \geq \underline{\theta}$. An equilibrium with $\theta^* = 1$ exists, whenever $y_2 \leq \overline{\theta}$. In an equilibrium with an intermediate threshold the proportion of attacking agents is given by

$$\theta^* = \frac{\theta^* + \underline{\theta} - y_1}{y_2 - y_1} \quad \Leftrightarrow \quad \theta^*(Y) = \frac{y_1 - \underline{\theta}}{1 + y_1 - y_2}.$$

An equilibrium with an intermediate threshold exists if and only if

$$0 < \theta^* < 1 \quad \Leftrightarrow \quad 0 < \frac{y_1 - \underline{\theta}}{1 + y_1 - y_2} < 1.$$

For $y_2 - y_1 < 1$ this condition is equivalent to $\underline{\theta} < y_1 < y_2 < \overline{\theta}$. For these public signals there exist multiple equilibria and $\theta^* = \frac{y_1 - \underline{\theta}}{1 + y_1 - y_2}$ is the third one besides those in which either all or no agent attacks.

For $y_2 - y_1 > 1$, an equilibrium with $0 < \theta^* < 1$ exists, if and only if $y_1 < \underline{\theta} < \overline{\theta} < y_2$, *i.e.* if the lower signal hints at a state, at which attacking is a dominant strategy and the high signal hints at a state where not-attacking is a dominant strategy. The existence of agents with different dominant strategies rules out equilibria in which all or no agent attack. The game has a unique equilibrium with an interior threshold that arises from the iterative elimination procedure as described above. The argument is illustrated in Figure 1: the bold line is the distribution of posterior beliefs F_Y as induced by a uniform distribution of the weights that agents attach to signals y_1 and y_2. It tells us how many agents have posterior beliefs below its argument. The steep line has slope 1 and should be viewed as an inverse threshold function, telling us up to which posterior belief agents attack the currency if they know that at least proportion θ of agents attacks. For agents with posteriors below $\underline{\theta}$ it is a dominant strategy to attack. Thus, there are at least $\theta_1 = F(\underline{\theta}) = (\underline{\theta} - y_1)/(y_2 - y_1)$ agents attacking. This raises the others' threshold to $\underline{\theta} + \theta_1$. Thereby, the proportion of attacking agents rises to at least $\theta_2 = F(\underline{\theta} + \theta_1)$, and so on. Since the distribution of posteriors F_Y has a slope smaller than 1, there is a unique equilibrium with an interior threshold and a proportion of attacking agents, given by the solution of $\theta^* = F(\underline{\theta} + \theta^*)$. If $y_1 < \underline{\theta}$ and $\underline{\theta} < y_2 < \overline{\theta}$, there is only one equilibrium, in which all agents attack. If $y_2 > \overline{\theta}$ and $\underline{\theta} < y_1 < \overline{\theta}$, there is a unique equilibrium, in which no agent attacks. Combining these results, multiple equilibria exist if and only if all signals are in the intermediate region.

Proposition 1: For a uniform distribution of subjective weights, the game with two public signals has multiple equilibria if and only if $\underline{\theta} < y_k < \overline{\theta}$ *for both k.*

This result shows that it makes a crucial difference, whether agents know the variances of public signals or not. For known variances, agents agree on the posterior and multiple equilibria exist, whenever this posterior is in $(\underline{\theta}, \overline{\theta})$. For unknown variances, multiplicity may require that all signals are in this region.

The simplicity of this result is due to the assumptions that the weights q are uniformly distributed and $K = 2$. However, it is not a general condition for multiplicity that all signals must be contained in the intermediate region. This can be seen by either assuming another distribution of weights or by considering more than two signals. For the case with $K = 2$, suppose that the distribution of subjective weights is uni-modal around 0.5. If the center of the interval $[y_1, y_2]$ is in the multiplicity region, there are less agents with posterior expectations in the dominance regions than for a uniform distribution. The cumulative distribution of posterior beliefs is steeper at the centre and may intersect the threshold function three times, which may give us multiple equilibria even if $y_1 < \underline{\theta} < \overline{\theta} < y_2$. An example is shown in Figure 2.

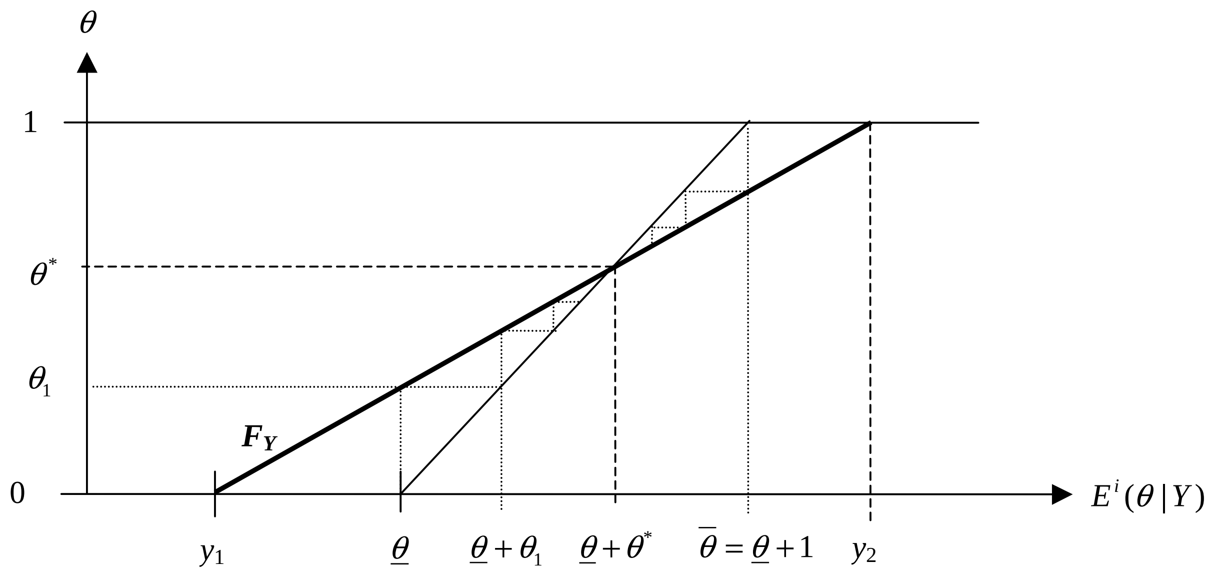

Figure 1. Iterative elimination of dominated strategies.

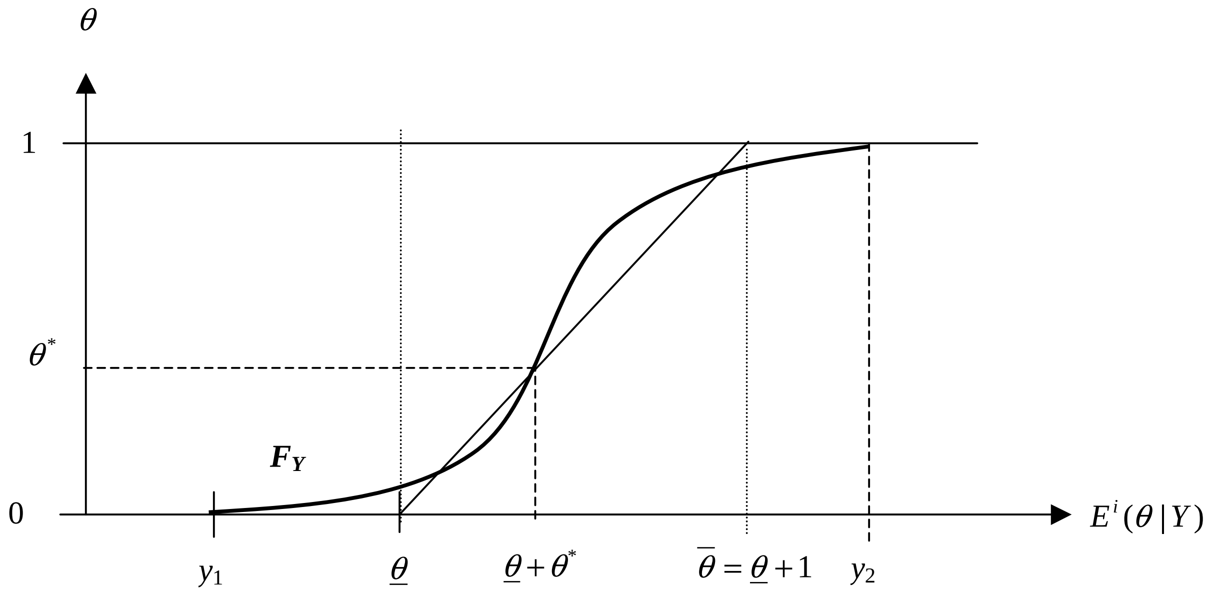

Figure 2. Multiple equilibria for a uni-modal distribution of posteriors.

3.2. Equilibrium in the case of three public signals

With more than two signals, we get a uni-modal distribution of posterior beliefs, even with a uniform distribution of q^i on the unit simplex. In order to get a unique equilibrium the range covered by the signals must increase. We demonstrate this for $K = 3$ and derive a sufficient condition for uniqueness of continuation equilibria.

From equation (3) it is clear that there is a unique equilibrium threshold if $f_Y(\theta) < 1$ for all θ. At the lowest and at the highest equilibrium, the derivative of the right hand side of (3) with respect to θ^* stays below 1. Multiplicity requires that there is an intermediate equilibrium, at which the cumulative distribution of posteriors rises faster than the threshold function. That is

$$f_Y\left(\theta^*(Y) + \underline{\theta}\right) > 1\,. \tag{4}$$

In the Appendix we show that $y_3 - y_1 > 2$ implies $f_Y(\theta) < 1$ for all θ. Hence, a sufficiently wide dispersion of the most extreme signals is a sufficient condition for uniqueness.

Proposition 2: For a uniform distribution of subjective weights, the game with three public signals has a unique equilibrium if $y_3 - y_1 > 2$.

For more than two public signals, uniqueness or multiplicity depend on the precise interaction between the distribution of signals and subjective weights. If all signals are close to each other and cover the intermediate region $(\underline{\theta}, \overline{\theta})$, then there are multiple equilibria, even for a uniform distribution of weights. However, if there is sufficient dispersion between the highest signal and the lowest signal, then uniqueness of the equilibrium is guaranteed independent from the range that is covered by these signals.

In particular, for $y_3 - y_1 > 2$, the slope of the cumulative distribution of posteriors is smaller than 1. Therefore, it can intersect the hurdle function between $\underline{\theta}$ and $\overline{\theta}$ at most once, so that there is a unique equilibrium. If the distribution of weights is more concentrated on the center of the simplex, then the extreme signals need to be even further away from each other to guarantee uniqueness.

The intuition behind this result is the following. If at least one signal is outside the region $(\underline{\theta}, \overline{\theta})$, the equilibrium may be unique and it can be derived by iterative elimination of dominated strategies. The iteration process starts with agents whose posteriors are such that either attacking or not-attacking is a dominated strategy. For the remaining agents with posteriors close to the edges of $(\underline{\theta}, \overline{\theta})$, either action looses its appeal, if they know that the proportion of attacking agents is bounded away from zero or one, respectively. This leads to a smaller region for which neither action is a dominant strategy. The size of these steps of elimination depends on the mass of agents for whom either action can be predicted from their extreme beliefs. If the number of agents with extreme beliefs is small, then the iteration procedures stop early and the interval for which posterior beliefs are self-fulfilling is reduced only slightly. However, if a sufficiently large mass of agents' posteriors is in the respective dominance region, the iteration steps are large and lead to a single threshold.

3.3. Equilibrium in the case of an infinite number of public signals

We determine the analytical solution for equilibrium uniqueness in the case where the number of public signals tends to infinity.

To ease the exposition, we assume $\tau_k^2 = \tau^2$ for all *k*. That is, all signals have the same precision. However, we keep the assumption that agents have private beliefs about these precisions. While the objective weights are $q_k = 1/K$ for all *k*, individuals attach private weights to the signals. When all signals have the same precision, the conditional variance of θ is $Var(\theta|Y) = \tau^2 / K$. The aggregate uncertainty after realization of signals becomes smaller with an increasing number of signals. With an infinite number of signals, $K \to \infty$, the uncertainty vanishes and agents are almost sure that their private posterior coincides with the true state θ. However, since agents differ in their evaluation of the various signals, they still disagree in their posterior beliefs. With $Var(\theta|Y) \to 0$, the range of posteriors for which there is no dominant strategy converges to the unit interval, $(\underline{\theta}, \overline{\theta}) \to (0,1)$.

Due to the law of large numbers, the distribution of realized signals is almost certainly identical to the prior distribution of signals, $y_k \sim N(\theta, \tau^2)$. However, the distribution of posterior beliefs, $E^i(\theta) = \sum_{k=1}^{\infty} q_k^i \, y_k$, depends also on the distribution of private weights $q^i \in \Delta^\infty$. Any distribution of weights induces a distribution of posteriors with probability one. Denote the cumulative density function of the distribution of posterior beliefs by *F*. The equilibrium condition (3) is then equivalent to $\theta^* = F(\theta^*)$.

Multiplicity of equilibria requires that there is a solution to this equation, at which $f(\theta^*) > 1$, where *f* is the non-cumulative density of posteriors. For a uniform distribution of weights on the simplex, the induced density of posteriors *f* has its maximum at the true state θ. The same is true for any single-peaked symmetric distribution of weights. This maximum decreases to zero with an increase in $\tau^2 \to \infty$. Hence, there is a critical level for the variance of public signals, such that for a higher variance there is a unique continuation equilibrium for almost all realizations of θ and *Y*. For lower variances, there may be multiple equilibria for some realizations of $\theta \in (0,1)$. An example is illustrated in Figure 3. The higher τ^2, the flatter is the distribution of posteriors.

Proposition 3: For any single-peaked symmetric distribution of weights on the simplex Δ^∞, *multiplicity of equilibria requires that* τ^2 *is sufficiently small.*

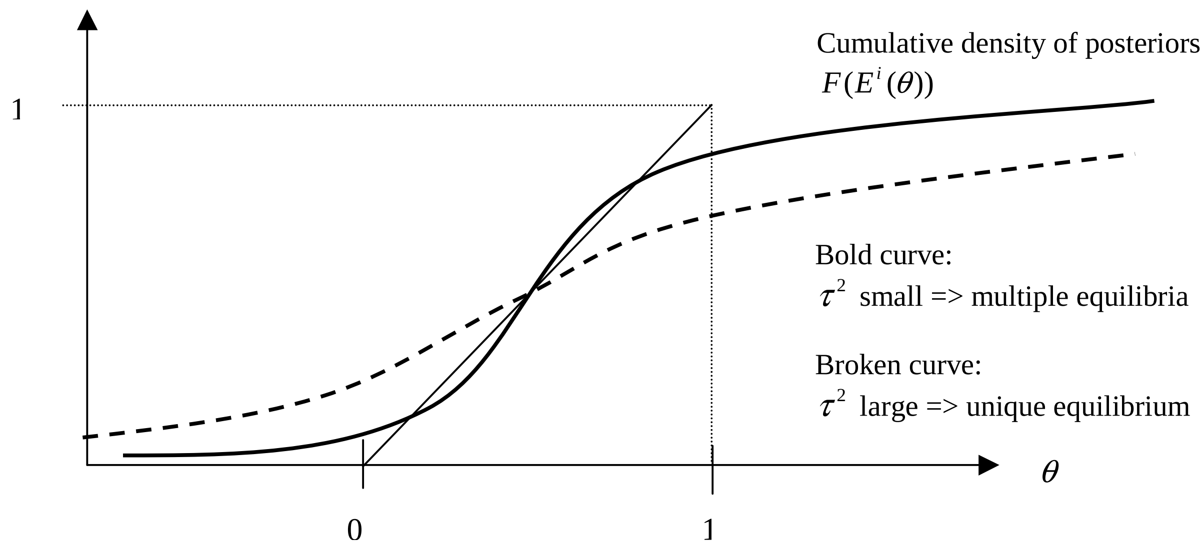

Figure 3. Multiple equilibria may exist for some $\theta \in (0,1)$, *if* τ^2 *is sufficiently small.*

For multiple equilibria, the cumulative distribution of posteriors must have a slope exceeding one (bold curve on Figure 3). Consider, for example that each agent attaches weight $q_k^i = 1$ to one of the signals, and assume that the aggregate distribution of weights is uniform on $k = 1, 2, \ldots, \infty$. Then, the posterior distribution of beliefs coincides with the distribution of signals. It is normal with mean θ and variance τ^2. The (non-cumulative) distribution is given by

$$f(E^i(\theta) \mid Y) = \phi\left(\frac{E^i(\theta) - \theta}{\tau}\right) = \frac{1}{\tau\sqrt{2\pi}} \exp\left(\frac{-(E^i(\theta) - \theta)^2}{2\tau^2}\right) \le \frac{1}{\tau\sqrt{2\pi}}.$$

Multiplicity requires that $f(\cdot) > 1$. Since f is bounded above, a necessary condition for multiple equilibria is $\tau < \frac{1}{\sqrt{2\pi}}$.

3.4. Intuition for *K* finite larger than 3

We now give some intuition for a large but finite number of signals. If public signals are rather precise, then most signals are close to the true state θ. Thereby, most agents' posteriors are close to the true state, even though these agents differ in their opinion about the relative precision of signals. If the true state happens to be in the interior of the region $(\underline{\theta}, \overline{\theta})$, then there are multiple equilibria for a sufficiently high precision of public signals. This occurs with some positive probability. The lower the precision of public signals, the wider the dispersion of posterior beliefs and the smaller is the region of states, for which multiple equilibria exist. Therefore, the probability that the economy is endangered by self-fulfilling beliefs gets smaller. If the precision of public signals is sufficiently small, then there is a unique continuation equilibrium with probability 1 for each state θ.

This shows that the precision of public signals is related to the prior probability of an economy being endangered by crises out of self-fulfilling beliefs. In this sense, our results lead to a similar conclusion as the global-game approach by Morris and Shin (2003): uniqueness requires that public information is not too precise, not to serve as a focal point generating common knowledge. However, our results differ from those by Morris and Shin, because we do not rely on the existence of rather precise private information. With pure public signals, uniqueness requires a sufficient dispersion of these signals *and* a sufficient dispersion of private beliefs about their relative importance. These private beliefs are common knowledge in our model: all agents know the distribution of these weights, and in an economy with a finite number of agents, our results would still hold, if agents know the actual weights of all other agents. Therefore, posterior beliefs are common knowledge, while uniqueness in the global-game approach requires that posterior beliefs are private information.

Nevertheless, the drawback of our approach is that equilibrium uniqueness requires a lot of disagreement about the fundamentals, while the global-games approach by Carlsson and van Damme

(1993) and its application to speculative attacks by Morris and Shin (1998, 2003) works best, if private information is rather precise and, thereby, posterior beliefs are close to each other. Uniqueness with multiple public signals requires that there are some agents who believe that one of the actions is a dominant strategy.[8] It requires the existence of at least some people who believe that the economy is in one of the extreme states.

4 – Simulation results for the prior probability of multiple continuation equilibria

Theoretical analysis has shown that multiplicity or uniqueness of continuation equilibria depends on the realized fundamental and signals as well as on the weights that private agents attach to these signals. Since these variables are random, there is always some probability of getting realizations with multiple continuation equilibria. The prior probability of getting uniqueness, however, depends on the distribution and on the number of signals. In this section we analyze the relationship between the number of signals and the prior probability of getting a unique continuation equilibrium. For $K \leq 2$ this probability can be calculated. For a larger number of signals, there is no algebraic solution allowing a precise computation. Instead, we employ a Monte Carlo simulation with random draws of states and signals for estimating the prior probability for a unique continuation equilibrium.

The following exogenous parameters are kept constant throughout each simulation:

t/R = relation between costs and potential gains from attacking,

θ_0 = *ex-ante* expected state,

σ^2 = variance of *ex-ante* distribution of θ.

Tables 1 and 2 display results for some of these parameters. Results for other parameter combinations and MATLAB programs for these simulations are available at

http://www.wm.tu-berlin.de/~makro/Heinemann/publics/multiple-public-info.html.

The number of public signals K is varied from 1 to 8. Continuation equilibria depend on the K signals. The distribution of signal y_k is normal around θ with variance τ_k^2. We assume symmetry in the sense that all signals have the same variance, $\tau_k^2 = \tau^2$. The conditional variance of the state given realized signals is then

$$V(\theta|Y) = \tau^2 / K . \tag{5}$$

We follow two routes with different interpretation:

[8] The global-game approach works, even when all agents know that the fundamental is in the intermediate region and know that others know that the fundamental is in the intermediate region, as long as this event is not common knowledge.

1. In a first set of simulations, we follow the idea that a central bank may release summary statistics of an economy or release disaggregate signals leaving it to the agents to aggregate them. Here, we adjust τ^2 to changes in *K*, such that the posterior variance of θ, given by equation (5) remains constant for changes in *K*. This implies that the information value of all signals together is always the same. We find that disaggregate signals lead to a lower probability for multiple continuation equilibria, as long as agents have different beliefs about the relative precision of these signals.
2. In a second set of simulations, we ask whether providing additional information by separate signals stabilizes the economy. Here, we keep τ^2 constant. This implies that the posterior variance $Var(\theta\,|\,Y)$ is decreasing in the number of signals *K*. We find that providing more signals tends to reduce the probability for multiple continuation equilibria. However, the results are less pronounced than in the first set of simulations.

For $K \le 2$, the prior probability of a unique continuation equilibrium can be calculated analytically. With a single public signal, *K* = 1, all agents have the same posterior $E^i(\theta\,|\,Y) = Y$. An attack can occur whenever $Y < \overline{\theta} = \underline{\theta} + 1$, with

$$\underline{\theta} = -\sqrt{Var(\theta\,|\,Y)}\,\Phi^{-1}(t/R).$$

Multiple equilibria exist, whenever $\underline{\theta} < Y < \overline{\theta}$. The prior probability of this event is

$$prob(\underline{\theta} < Y < \overline{\theta}\,|\,\theta_0) = \Phi\left(\frac{\overline{\theta}-\theta_0}{\sqrt{\sigma^2+\tau^2}}\right) - \Phi\left(\frac{\underline{\theta}-\theta_0}{\sqrt{\sigma^2+\tau^2}}\right).$$

For *K* = 2, we assume a uniform distribution of weights $q_i \in \Delta^2$. From Proposition 1 we know that multiple equilibria exist, if and only if both signals are in $(\overline{\theta},\underline{\theta})$. The prior probability of this event is

$$\text{prob}\left(y_1 \in (\overline{\theta},\underline{\theta}) \ \wedge\ y_2 \in (\overline{\theta},\underline{\theta}) \,\middle|\, \theta_0\right)$$

$$= \int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{2\pi\sqrt{\tau^4+2\sigma^2\tau^2}} \exp\left(\frac{-\sigma^2(y_1-y_2)^2+\tau^2\left((y_1-\theta_0)^2+(y_2-\theta_0)^2\right)}{2\left(\tau^4+2\sigma^2\tau^2\right)}\right) dy_1\, dy_2\,,$$

where y_1 and y_2 are joint normal with prior expected value θ_0, variance $\sigma^2+\tau^2$ and covariance σ^2.

For $K \ge 2$, we use simulations to estimate the probability for multiple continuation equilibria[9]. In simulations we must consider a finite number of agents. We approximate the uniform distribution of weights by using one agent for each knot on a grid of the unit simplex. Agent *i* is now defined by

[9] The case *K* = 2 serves for controlling the deviation of simulation results from the theoretical value.

$\left(q_{i,k}\right)_{k=1,\ldots,K}$ and associated with a posterior belief

$$E^i\left(\theta \middle| Y\right) = \sum_k q_k^i \; y_k \; .$$

Grid size *d* is varied with *K*, for getting a better approximation of the uniform distribution for low values of *K* and a computable number of agents for large *K*. The grid size is 0.005 for *K* = 2, giving us 201 agents with different weights. For *K* = 3, we use a grid of size 0.05, giving us 231 agents, for *K* = 4, we use *d* = 0.1, giving us 286 agents. For *K* =5, 6, 7, and 8, we use *d* = 0.2, giving us 126, 252, 462, and 792 agents, respectively.

For each combination of parameters t/R, θ_0, σ^2, τ^2, and *K*, we draw 10,000 random combinations of $(\theta, y_1, \ldots, y_K)$. First, we draw θ according to the normal distribution around θ_0, then the signals conditional on the realization of θ. Then, we search for continuation equilibria, that are characterized by thresholds $\theta^*(Y)$, solving the following equation:

$$\theta^*(Y) = \frac{1}{N(K)} \left| \left\{ i \,\middle|\, E^i(\theta \,|\, Y) < \theta^*(Y) + \underline{\theta} \right\} \right| . \tag{6}$$

The proportion of random draws, for which there are multiple continuation equilibria serves as an estimate of the prior probability that the game has multiple continuation equilibria.

In total, we ran calculations (for $K \in \{1, 2\}$) and simulations for all combinations of exogenous parameters defined by $t/R \in \{0.1, 0.25, 0.5, 0.75\}$, $\theta_0 \in \{-0.1, 0, 0.5, 1, 2\}$, $\sigma \in \{0.05, 0.2, 0.5, 1, 2\}$, and $Var(\theta \,|\, Y) \in \{0.05, 0.2, 0.5, 1, 2\}$. This gives 500 different combinations. The average probabilities for multiplicity over all 500 cases are displayed in the last line of Table 1. In all cases, the probability for multiple continuation equilibria for $K = 2$ is smaller than for $K = 1$. When going from simulations with *K* = 2 to *K* = 4, the probability for multiple equilibria is decreasing in all but one case. Comparing simulations for *K* = 4 with those for *K* = 8, we find that in 8% of all cases the probability for multiple equilibria is higher for *K* = 8. The difference is always less then 0.8% and occurs only in cases, where this probability is already below 4% for *K* = 4. We conclude that, in general, the prior probability of multiple continuation equilibria is lower if disaggregate signals are provided, as long as agents have different beliefs about the relative precisions of these signals.

Exogenous parameters				Probability of multiple continuation equilibria									
t/R	θ_0	σ	$V(\theta \vert Y)$	*K*=1	*K*=2 analyt.	*K*=2 simul.	*K*=3	*K*=4	*K*=5	*K*=6	*K*=7	*K*=8
0.1	-0.1	0.2	0.05	9.9	4.1	4.1	1.9	1.2	0.7	0.5	0.4	0.3
0.1	-0.1	1	0.05	26.5	17.2	14.6	9.9	7.2	6.8	5.7	5.6	4.3
0.1	-0.1	1	0.2	20.6	8.3	8.4	5.6	4.0	3.1	2.8	3.0	3.1
0.1	0.5	0.2	0.05	75.7	51.6	51.2	37.2	29.6	12.2	8.4	5.8	4.8
0.1	0.5	1	0.05	36.1	23.7	23.0	16.9	13.9	9.8	8.2	7.1	6.2
0.1	0.5	1	0.2	31.0	12.7	12.5	10.4	8.0	5.5	4.8	4.7	5.2
0.1	1	0.2	0.05	82.2	58.3	57.2	42.9	35.4	25.7	22.6	19.5	17.2
0.1	1	1	0.05	36.7	24.1	24.1	17.0	13.5	9.3	8.4	7.2	6.5
0.1	1	1	0.2	35.1	14.5	14.6	9.1	7.0	5.7	4.8	5.3	5.8
0.1	2	0.2	0.05	0.9	0.3	0.3	0.1	0.04	0.01	0.03	0.05	0. 03
0.1	2	1	0.05	19.6	12.6	11.8	9.4	6.8	5.3	4.2	3.6	3.1
0.1	2	1	0.2	24.2	10.3	10.9	6.0	5.2	4.0	3.6	3.8	4.0
0.5	-0.1	0.2	0.05	36.9	19.7	20.2	12.3	8.8	6.0	5.0	4.1	3.2
0.5	-0.1	1	0.05	32.0	20.9	20.9	14.6	12.3	8.7	7.4	6.3	5.7
0.5	-0.1	1	0.2	30.6	12.5	12.7	7.8	5.8	5.2	5.0	4.7	5.0
0.5	0.5	0.2	0.05	90.4	68.8	67.4	52.9	44.1	33.3	18.2	13.9	11.7
0.5	0.5	1	0.05	38.4	24.6	24.1	17.0	13.9	10.6	8.3	7.7	6.8
0.5	0.5	1	0.2	35.2	14.5	14.1	9.2	6.8	5.0	5.6	4.8	5.2
0.5	1	0.2	0.05	50.0	29.0	29.1	19.3	13.6	9.9	7.9	7.0	6.0
0.5	1	1	0.05	33.5	22.0	22.1	15.4	13.7	8.7	7.5	6.5	5.9
0.5	1	1	0.2	31.9	13.1	13.5	8.0	6.3	5.3	4.8	5.5	5.3
0.5	2	0.2	0.05	< 0.1	< 0.1	0	0	0	0	0	0	0
0.5	2	1	0.05	13.9	8.9	8.7	6.1	4.9	3.8	3.0	2.6	2.3
0.5	2	1	0.2	14.7	5.9	5.7	3.5	2.8	2.0	2.2	2.4	2.2
average over 500 cases				26.9	10.0	10.3	7.0	5.8	4.3	3.9	3.8	3.9

Table 1. Probability of multiple continuation equilibria for constant posterior variance in per cent.

Following the second interpretation, we keep τ constant. Then, the probability that two signals fall in the region $(\underline{\theta}, \overline{\theta})$ is smaller than the probability that a single public signals falls into this region. Therefore, the probability of getting multiple continuation equilibria is smaller for $K = 2$ than for $K = 1$. Combining the values for t/R, θ_0, and σ as described above with $\tau \in \{0.01, 0.05, 0.2, 0.5, 1\}$, we have once more 500 parameter combinations. When going from simulations with *K* = 2 to *K* = 4, the probability for multiple equilibria is decreasing in about 92% of all cases. Going from K = 4 to K = 8, we find an increase in the probability for multiple equilibria in 13% of all cases, but we cannot relate these cases to a particular high or low probability at K = 4. We conclude that for constant τ^2, there is a tendency for more signals reducing the probability of multiple equilibria, but it is less pronounced and possibly less robust than for a constant posterior variance.

Exogenous parameters				Probability of multiple continuation equilibria					
t/R	θ_0	σ	τ	K=1	K=2 analyt.	K=2 simul.	K=3	K=4	K=8
0.1	-0.1	0.2	0.05	24.0	19.8	20.5	18.9	18.5	16.9
0.1	-0.1	1	0.05	31.6	29.9	30.8	28.6	28.8	27.9
0.1	-0.1	1	0.2	28.7	22.4	22.8	20.7	20.0	19.4
0.1	0.5	0.2	0.05	98.2	97.6	97.4	97.3	97.4	97.1
0.1	0.5	1	0.05	38.2	36.2	36.7	35.6	34.5	33.9
0.1	0.5	1	0.2	37.1	29.3	29.5	26.1	25.8	23.2
0.1	1	0.2	0.05	58.7	53.3	52.2	48.3	46.4	42.5
0.1	1	1	0.05	34.8	32.9	32.7	32.1	30.9	30.6
0.1	1	1	0.2	36.0	28.4	28.8	25.0	23.7	21.5
0.1	2	0.2	0.05	< 0.1	< 0.1	0	0	0	0
0.1	2	1	0.05	14.5	13.6	13.6	12.2	12.1	12.5
0.1	2	1	0.2	17.4	13.3	13.1	11.1	9.7	9.2
0.5	-0.1	0.2	0.05	31.4	26.5	26.3	24.6	22.7	21.0
0.5	-0.1	1	0.05	32.4	30.7	31.1	29.5	29.4	29.3
0.5	-0.1	1	0.2	32.1	25.2	24.7	21.7	22.0	19.4
0.5	0.5	0.2	0.05	98.5	97.9	98.1	97.7	97.4	97.2
0.5	0.5	1	0.05	38.2	36.3	36.7	34.8	34.1	33.2
0.5	0.5	1	0.2	37.6	29.8	29.5	26.6	24.7	23.3
0.5	1	0.2	0.05	50.0	44.5	43.0	40.7	40.1	37.8
0.5	1	1	0.05	34.1	32.3	31.8	31.0	31.3	29.8
0.5	1	1	0.2	33.7	26.5	26.2	22.5	22.7	20.8
0.5	2	0.2	0.05	< 0.1	< 0.1	0	0	0	0
0.5	2	1	0.05	13.6	12.8	12.9	12.1	11.7	12.3
0.5	2	1	0.2	14.2	10.5	10.8	9.3	8.2	7.9
average over 500 cases				33.4	23,6	23.5	21.5	20.6	19.2

Table 2. Probability of multiple continuation equilibria for constant variance of signals in per cent.

Comparing the numerical effects from Tables 1 and 2, increasing the number of signals, while keeping the variance of each signal constant (Table 2), tends to reduce the probability of getting multiple continuation equilibria to a lower extent than for a constant variance of posterior information

(Table 1), even when going from $K = 1$ to $K = 2$. When τ^2 is kept constant, there are two effects going in opposite directions: dispersed weights on an increasing number of signals stabilize the economy, a lower posterior variance associated with more signals destabilizes the economy. Our simulations indicate that in most but not all cases the first effect is stronger and additional informative signals reduce the probability of getting multiple continuation equilibria. Using the insights from our theoretical analyses, we expect that the stabilizing effect gets smaller, if subjective weights have a uni-modal instead of a uniform distribution. For the same reason, a uni-modal distribution should lead to smaller numerical effects of the number of signals on the probability for multiple equilibria if the posterior variance is kept constant.

There are two ways by which the stability of the economy may be endangered. First, the economy may be prone to self-fulfilling crises; to avoid this, the policy maker aims at reducing the probability of multiple continuation equilibria. Second, the economic outcome is a unique equilibrium with attack. The policy maker would like to avoid both kinds of instability. However, these two goals may be conflicting: multiple public signals might reduce the probability of multiple continuation equilibria at the expense of reducing the probability for a unique equilibrium without crisis.

To investigate this question, we take a particular selection of exogenous variables ($t/R = 0.5$, $\sigma = 1$, $Var(\theta \mid Y) = 0.2$, and $\theta_0 \in [-3, 4]$), where θ_0 is varied in steps of 0.05. For $K = 1$ and $K = 2$ we calculate the probabilities for a unique continuation equilibrium with all agents attacking, for a unique continuation equilibrium with no attack, and for multiple continuation equilibria. For $K = 2$ there are two additional cases to consider: if one signal is smaller than $\underline{\theta}$ and the other is larger than $\overline{\theta}$, then there is a unique equilibrium with some but not all agents attacking. The attack may be successful or not, depending on the realization of θ. Figures 4 and 5 show how the probabilities for these continuation equilibria depend on θ_0. Comparing both figures indicates that for $K = 2$ the probabilities for unique continuation equilibria with and without crisis are both higher than for $K = 1$.

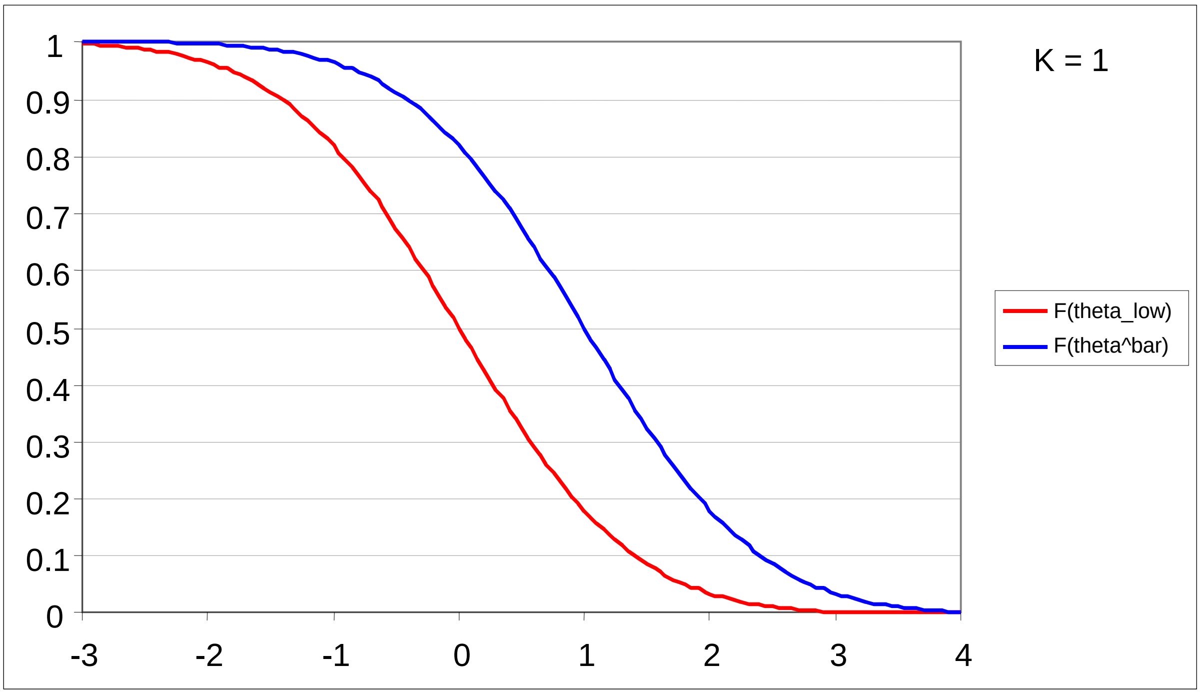

Figure 4: K = 1. For each value of θ_0, the lower solid curve indicates the probability for a unique equilibrium with all agents attacking, the upper solid curve is 1 minus the probability for a unique equilibrium without attack. The distance between the two curves indicates the probability for multiple equilibria.

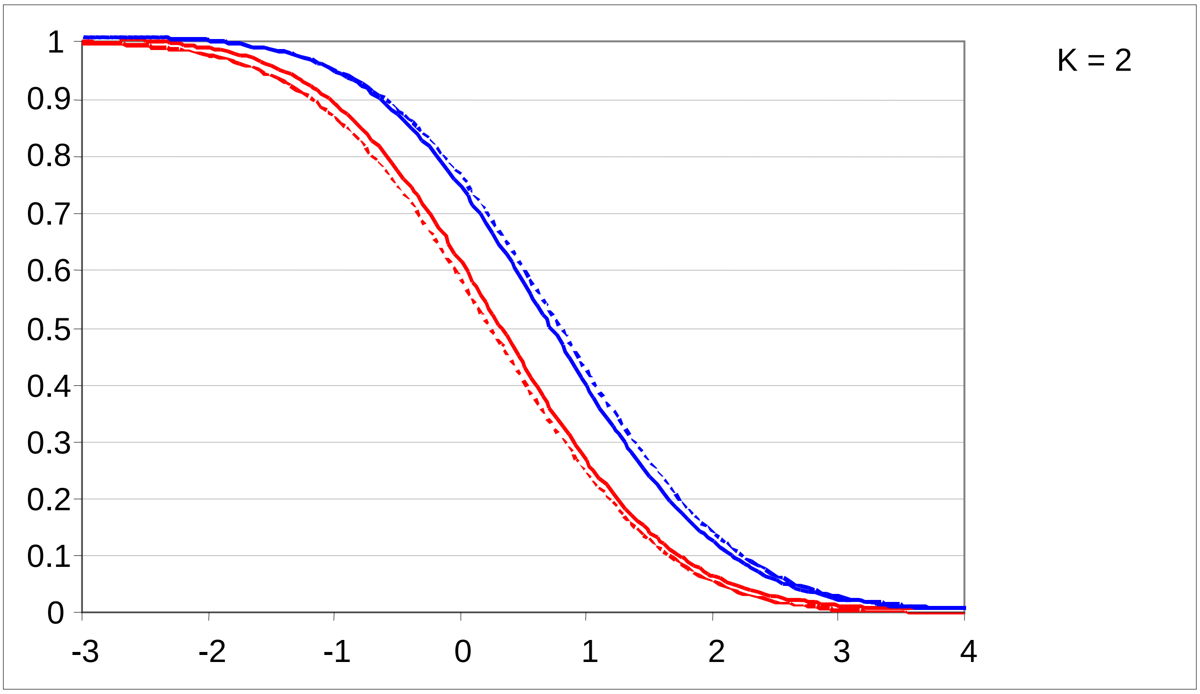

Figure 5: K = 2. For each value of θ_0, the lower solid curve indicates the probability for a unique equilibrium in which attacks are successful, the upper solid curve is 1 minus the probability for a unique equilibrium without successful attack. The distance between the two curves indicates the probability for multiple equilibria. The lower broken curves indicate the probability for an equilibrium in which all agents attack, the upper broken curve is 1 minus the probability for a unique equilibrium in which no agent attacks.

5 – The effect of public announcements: some implications for economic policy

Our analysis of the currency-attack model with multiple public signals has some consequences in terms of economic and informational policies. The model contributes to the current debate on the effects of reinforced transparency. Indeed, the fact that central banks and newspapers release information publicly raises concerns of whether economic transparency may be destabilising by rendering the economy prone to self-fulfilling crashes. Nevertheless, our model suggests a counterargument to the traditional view: agents deal the same information differently and posterior beliefs may differ even if all information is publicly disclosed, as soon as there are multiple public signals of unknown precision.

We first evoke the policy instruments derived from the model, especially the role of signals' precision and the number of announcements. We then illustrate our results with some considerations about the committee structure of central banks.

5.1. Policy instruments within the model: precision of signals, uncertainty on precision and number of announcements

Policy instruments affect economic stability via their impact on (i) the probability of getting multiple continuation equilibria and (ii) the range of probabilities with which a currency attack occurs, given appropriate beliefs in cases of multiple continuation equilibria.

5.1.1. Reducing the probability of multiple equilibria

We have shown that there may be a unique equilibrium if there is at least one public signal that hints at a state at which either attacking or not-attacking is a dominant strategy. With multiple public signals, multiplicity of equilibria requires that (i) signals are not too dispersed and (ii) private beliefs about the relative precision of these signals do not differ too much. These two conditions interact: if signals are dispersed over a wide range, there may still be multiple equilibria if most agents attach high weights to the same signals and *vice versa*: even if agents differ considerably in their beliefs about signals' precisions, there may be multiple equilibria, if most signals are concentrated in a critical region.

The probability of the economy being prone to self-fulfilling beliefs depends on the average precision of signals. If a large number of public signals gets very precise, we approach the case with perfect information. The lower the precision of public signals, the smaller gets the set of states with multiple equilibria and the smaller is the prior probability that the true state falls in this region. For a sufficiently low precision, the equilibrium is always unique, if the number of signals is large.

The economic policy implications of these results require distinguishing three dimensions of transparency: the number of public signals, their precision, and information about the precision of statements, for example reliable figures on expected forecast errors. A larger number of signals (or more frequent provision of information) helps to avoid overreactions to any single announcement. A higher precision is useful for markets in determining the consequences of actions. But, it also raises the *ex-ante* probability of getting multiple continuation equilibria. Finally, if agents agree on the precision of signals, their posteriors coincide, which leads to the same effects as providing a summary statistic as a single public signal. It induces high common weights to the announcements that may lead to crises out of self-fulfilling beliefs if the common posterior indicates a critical situation.

Agents do not always over-react to public information. Indeed, when there are multiple public signals – whose precisions are not common knowledge – the probability that realizations of random variables fall in a range with multiple equilibria is lower than for a single public signal (this issue has been extensively discussed in Section 4). The economy is less likely prone to crises out of self-fulfilling beliefs. As a consequence, the economy is relatively more stable with $K > 1$. This gives a role to the precision of signals: apart from its degree, uncertainty on it can represent an effective tool for the central bank to control for the beliefs of the agents. It may be rational for central banks not to publish forecast errors. The number of signals is also essential; having just two (appropriate) public signals in the market instead of one can already lead to a significant reduction in the probability for self-fulfilling-belief equilibria.

Whether additional signals are helpful (in terms of stabilisation), does not just depend on their number, but also on their realized values. For example, if the new disclosed signals accumulate in the intermediate region, it can be worse for the central bank to give more announcements. The content of announcements is also very important. Suppose agents receive two public signals. If some additional announcements (say two) cross either border, the set of equilibria may switch to another regime, as represented in Figure 6. We thus make the case that public information is not *per se* (automatically) destabilising. Our model is less deterministic than second generation models that always give multiple equilibria in the intermediate zone and private information models that always find some conditions for uniqueness (as soon as private information is sufficiently precise). Providing multiple public signals does not exclude multiple equilibria, but reduces the likelihood that conditions for multiplicity are met.

Initial situation with two public signals: unique equilibrium → predictable outcome

Situation after providing two more signals y_3 and y_4: multiple equilibria → unpredictable outcome

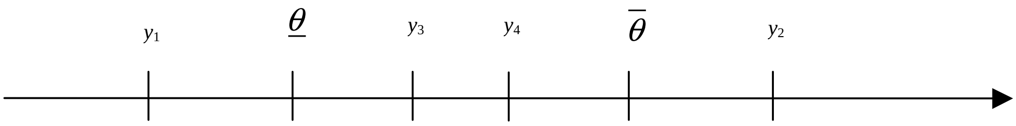

Figure 6. Destabilization by Public Announcements.

5.1.2. Increasing the probability of getting a unique equilibrium without successful attack

As we have seen in Section 4, when agents have private beliefs about the precision of the signals they receive, providing aggregate or disaggregate information may also affect the relative probability between getting a unique equilibrium with and without financial crisis. For example, comparing Figures 4 and 5 indicates that for $K = 2$ the probabilities for unique continuation equilibria with and without crisis are both higher than for $K = 1$. A larger number of signals may therefore increase the probability of attack.

The effect of a higher precision is useful for markets in determining the consequences of actions. It contributes to reinforce market discipline. So when fundamentals are rather good, information of higher quality reduces the probability of a successful attack and conversely. And finally, if agents tend to agree on the precision of signals, again, the outcome depends on the realized fundamental value.

Overall, depending on which welfare criterion is chosen (increasing market discipline *vs.* reducing the probability of getting a unique equilibrium with crisis), our results suggest that it might be good for a policy maker to speak with many voices. Nevertheless, although the model does not directly deal with the associated costs of the many voices approach, following Fracasso, Genberg and Wyplosz (2003), we can mention some of the costs generated by such a policy. To achieve legitimacy, accountability and credibility, the policy maker (very often the central bank) must convince the public that it has the expertise to achieve its goals. Conflicting or contradictory signals may impede such understanding from the economic agents. Too much information may kill information, because of the possible limited capacity of economic agents to treat and aggregate a very large number of news.

Many signals may be confusing: the more fragmented information is, the less likely it is that the whole picture may be well understood by agents, because not all fragments are recognized by all agents. As Morris and Shin (2007, p. 599) put it, "*when a central bank relies on a myriad of speeches and testimonies […], even if the collection of speeches taken together convey a coherent message, the fragmented nature of the communication leaves open the possibility that some market observers […] fail to capture the intended picture, with its subtle emphases and qualifications*". More fragmented information goes hand in hand with a greater risk for miscommunication. By multiplying the information, a central bank sends different messages to different constituencies, then using communication to obfuscate not to enhance transparency. Looking for information may also be financially costly and time-consuming (cost of information acquisition); when information is fragmented and dispersed these costs are particularly important.[10]

5.2. Illustration: communication policy of central banks and committee structure

One of the major recent trends in central banking practice has been the formal adoption of decision-making by Monetary Policy Committees (MPCs) rather than by individual central bank heads.[11] However, committee structures remain highly various. Blinder *et al.* (2001) provide a detailed typology of these committees. They especially distinguish collegial committees, where decision is taken by consensus[12] from individualistic MPCs (*e.g.*, the Bank of England), where each member not only expresses his opinion verbally but also acts by voting; in such a case, unanimity is not necessary.

Which committee structure is better suited for the conduct of monetary policy? This issue has recently been under discussion. In an experiment, Blinder and Morgan (2004) argue that diversification pays off in the form of better decisions. Therefore an individualistic committee, which takes full advantage of the committee's diversity, would seem to have a clear edge over a collegial committee, which exploits diversity much less. However, as pointed by Blinder and Wyplosz (2005, p.11), several voices potentially create confusion: "*The danger arises if an individualistic committee is undisciplined and speaks with too many voices, especially if those disparate voices carry conflicting messages. In that case, central bank transparency can degenerate into central bank cacophony, leaving outside observers more befuddled than enlightened.*" One argument that goes against such an argumentation is that the talk that emanates from the Bank of England's MPC, and which is the expression of many voices[13], does seem to inform markets much more than it confuses them.

[10] Some central banks, like the Bank of France, charge local actors for sector-specific or regional data that are accompanied by an analysis of the central bank.
[11] See Blinder *et al.* (2001) and Blinder and Wyplosz (2005) for reviews on this issue.
[12] There is also a variety of collegial committees, with two polar cases being the "autocratically-collegial MPC" where the chairman dictates the consensus and the "genuinely collegial MPC" where members argue for their own points but finally compromise on a group decision. The Federal Reserve System is a good example of the former, while the European System of Central Banks can represent the latter.
[13] Indeed, the Bank of England publishes *the minutes* (of MPC discussions) where differences in opinions are an essential part of the information that should be conveyed to the markets.

Our framework provides some useful insights to this debate arguing that several point of views expressed by members of a committee could be stabilizing as market participants may not all favor the information given by the same member and rather have different preferences about who to believe in. It therefore makes a case for an individualistic monetary policy committee in economies that may be threatened by self-fulfilling prophecies like in particular emerging market economies.

6 – Conclusion

This paper sheds light on the difficulties linked with the dichotomy between public information on the one hand and private information on the other. The literature on global games has realized that markets might be more stable if publicly provided information is associated with private beliefs. However, the literature assumes that distribution functions are common knowledge. With this assumption, public signals induce public beliefs. Instead, we argue that distribution parameters may be private information. Then, public signals induce idiosyncratic beliefs. Diverse sources of information or differences in the treatment of information can be responsible for the idiosyncratic weights on public signals by which common posteriors are avoided. Thus creating sufficient differences in the evaluation of publicly available information contributes to preventing self-fulfilling beliefs equilibria. Here, we try to fill in a theoretical gap between public and private information, by proposing a private value game applied to the traditional speculative-attack model.

It is well known that common knowledge is difficult to establish in practice. However, financial markets are very transparent and many informational signals are disclosed by the central bank, or any other institution. On the exchange rate market, there is a plurality of channels (*media*), which disclose more or less precise (but “objectively mistaken”) public information. Central bank committee members themselves can sometimes express diverging views about the conduct of monetary policy. Hence, rational agents are aware that any public information is observed by all other agents. However, common knowledge of *posterior* beliefs does not only require that all agents share the same information, it also requires that agents share the beliefs about the conditional distribution of the revealed information, given the fundamentals. As a consequence, even if all agents share the same information, agents may differ in their evaluation of these signals, and thus in their posterior beliefs. This does not require private information, although different beliefs about precisions may be viewed at as resulting from private information. Agents may even agree to disagree. By creating disparities between agents’ posterior beliefs, multiple sources of public information can avoid self-fulfilling beliefs equilibria. Such a model can help to explain why and how attacks are determined, even when the most relevant information about fundamentals is publicly disclosed.

7 – References

ANGELETOS G.-M., WERNING I. [2006], "Crises and prices – information aggregation, multiplicity and volatility," *American Economic Review*, 96(5), pp. 1720-1736.

AUMANN R. [1976], "Agreeing to Disagree," *The Annals of Statistics*, 4 (6), pp. 1236-1239.

BANNIER C., HEINEMANN F. [2005], "Optimal Transparency and Risk Taking to Avoid Currency and Liquidity Crises," *Journal of Institutional and Theoretical Economics*, 161 (3), pp. 374-391.

BLINDER A.S., GOODHART C., HILDEBRAND P., LIPTON D., WYPLOSZ C. [2001], "How Do Central Bank Talk?," *Geneva Reports on the World Economy*, 3, Geneva: International Center for Monetary and Banking Studies.

BLINDER A.S., MORGAN J. [2005], "Are Two Heads Better than One?: Monetary Policy by Committee," *Journal of Money, Credit, and Banking*, 37(5), pp. 789-812.

BLINDER A.S., WYPLOSZ C. [2005], "Central Bank Talk: Committee Structure and Communication Policy," paper prepared for the session "Central Bank Communication" at the ASSA meetings, Philadelphia, January 9, 2005.

CARLSSON H., VAN DAMME E. [1993], "Global Games and Equilibrium Selection," *Econometrica*, 61(5), pp. 989-1018.

DÖNGES J., HEINEMANN F. [2006], "Competition for Order Flow as a Coordination Game," http://www.wm.tu-berlin.de/~makro/Heinemann/download/CN.pdf.

FRACASSO A., GENBERG H., WYPLOSZ C. [2003], "How Do Central Banks Write?," *Geneva Special Report on the World Economy*, 2, Geneva: International Center for Monetary and Banking Studies.

HEINEMANN F., ILLING G. [2002], "Speculative Attacks: Unique Sunspot Equilibrium and Transparency," *Journal of International Economics*, 58(2), pp. 429-450.

HELLWIG C. [2002], "Public Information, Private Information and the Multiplicity of Equilibria in Coordination Games," *Journal of Economic Theory*, 107, pp. 191-222.

HELLWIG C., MUKHERJI A., TSYVINSKI A. [2006], "Self-fulfilling currency crises: The role of interest rates," *American Economic Review*, 96(5), pp. 1769-1787.

HELLWIG C., VELDKAMP L. [2007], "Knowing What Others Know: Coordination Motives in Information Acquisition," *forthcoming in Review of Economic Studies*.

LINDNER A. [2006], "Does Transparency of Central Banks Produce Multiple Equilibria on Currency Markets," *Scandinavian Journal of Economics*, 108(1), pp. 1-14.

METZ C. [2002], "Private and Public Information in Self-Fulfilling Currency Crises," *Journal of Economics*, 76(1), pp. 65-85.

MORRIS S., SHIN H.S. [1998], "Unique Equilibrium in a Model of Self-fulfilling Currency Attacks," *American Economic Review*, 88(3), pp. 587-597.

MORRIS S., SHIN H.S. [2003], "Global Games: Theory and Applications," in Mathias Dewatripont, Lars Peter Hansen and Stephen J. Turnovsky (eds), *Advances in Economics and Econometrics: Theory and Applications*, Volume One (pp.56-114), Cambridge University Press, London.

MORRIS S., SHIN H.S. [2007], "Optimal communication," *Journal of the European Economic Association*, 5 (proceedings), pp. 594-602.

MYATT D., SHIN H.S., WALLACE C. [2002], "Games and Coordination," *Oxford Review of Economic Policy*, 18(4), pp. 1-20.

OBSTFELD M. [1986], "Rational and Self-Fulfilling Balance-of-Payments Crises," *American Economic Review*, 76(1), pp. 72-81.

OBSTFELD M. [1996], "Models of Currency with Self-fulfilling Features," *European Economic Review*, 40, pp. 1037-1047.

OZDENOREN E., YUAN K. [2008], “Feedback Effects and Asset Prices,” forthcoming in *Journal of Finance*.

ROMER C., ROMER D. [2000], “Federal Reserve Private Information and the Behavior of Interest Rates,” *American Economic Review*, 90(3), pp.429-457.

8 – Appendix: PROOF of *Proposition 2*

First, note that $\left|\Delta^3\right| = \sqrt{3}/2$. For $K = 3$ and uniform distribution of weights, equilibrium condition (3) is equivalent to

$$\theta^* = \frac{2}{\sqrt{3}}\left|\left\{q \in \Delta^3 \,\middle|\, q_1 y_1 + q_2 y_2 + q_3 y_3 \le \theta^* + \underline{\theta}\right\}\right|. \quad (4)$$

With a uniform distribution of weights on the simplex, there is a positive mass of agents, who believe strongly in the worst signal and a positive mass of agents who believe strongly in the best signal. An equilibrium with $\theta^* = 1$, in which all agents attack, exists, if and only if $y_k \le \overline{\theta}$ for all *k*. An equilibrium with $\theta^* = 0$, in which no agent attacks, exists if and only if $y_k \ge \underline{\theta}$ for all *k*.

Multiple equilibria require the existence of at least one interior equilibrium, $0 < \theta^* < 1$, at which the derivative of the right hand side of (4) with respect to θ^* exceeds 1.

For any equilibrium with interior threshold,

$$\theta^* = \frac{2}{\sqrt{3}}\left|\left\{q \in \Delta^3 \,\middle|\, q_1 y_1 + q_2 y_2 + q_3 y_3 = \theta^* + \underline{\theta}\right\}\right|. \quad (5)$$

Hence, an interior equilibrium requires that $y_1 < \theta^* + \underline{\theta} < y_3$. So, there exists a linear combination of y_1 and y_3 with $(1 - q_A)y_1 + q_A y_3 = \theta^* + \underline{\theta}$, which is equivalent to $q_A = \dfrac{\theta^* + \underline{\theta} - y_1}{y_3 - y_1}$.

In Figures 7a and 7b this point is given by A. Now we distinguish two cases: If $y_2 \ge \theta^* + \underline{\theta}$, then there also exists a linear combination of y_1 and y_2 that equals $\theta^* + \underline{\theta}$. This is indicated by point B in Figure 7a. Any combination of weights on the straight line between A and B is associated with the same expected state. In an equilibrium of this type, the area on the simplex below the line AB divided by the total size of the simplex equals θ^*.

If $y_2 < \theta^* + \underline{\theta}$, then there exists a linear combination of y_2 and y_3 that equals $\theta^* + \underline{\theta}$. This is indicated by point B in Figure 7b. Again, any combination of weights on the straight line between A and B is associated with the same expected state. In an equilibrium of this type, θ^* equals the area on the simplex below the line AB divided by the total size of the simplex.

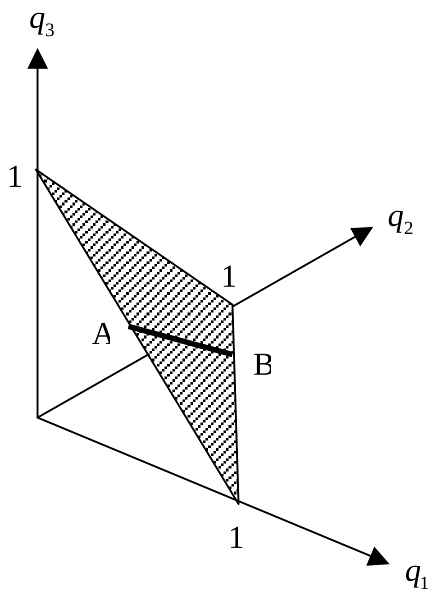

Figure 7a.

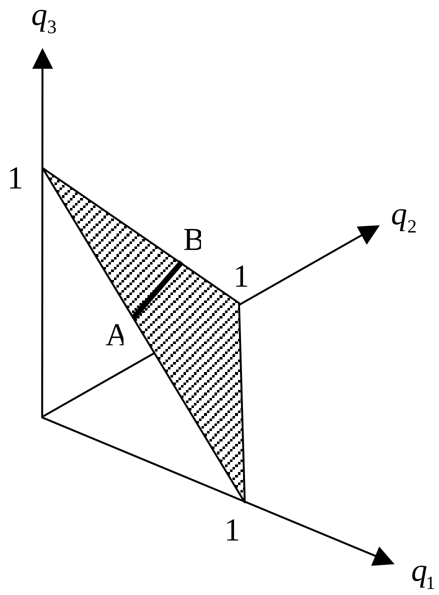

Figure 7b.

In both figures the shaded area is the unit simplex. Points on the simplex below AB *are associated with posterior expectations* $\sum_{k=1}^{3} q_k y_k \leq \theta^* + \underline{\theta}$.

If $y_2 \geq \theta^* + \underline{\theta}$, the coordinates of B are $\left(\frac{y_2 - \theta^* - \underline{\theta}}{y_2 - y_1}, \frac{\theta^* + \underline{\theta} - y_1}{y_2 - y_1}, 0 \right)$. Basic rules of trigonometry enable us to calculate the area below AB; It has the size $\frac{\sqrt{3}}{2} \cdot \frac{(\theta^* + \underline{\theta} - y_1)^2}{(y_3 - y_1)(y_2 - y_1)}$. Thus, the condition for an interior equilibrium threshold (5) is equivalent to $\theta^* = \frac{(\theta^* + \underline{\theta} - y_1)^2}{(y_3 - y_1)(y_2 - y_1)}$. The right-hand side is increasing and concave in θ^*. So, the derivative of the right-hand side is maximal at the highest θ^* at which the condition $y_2 \geq \theta^* + \underline{\theta}$ applies, *i.e.* at $\theta^* = y_2 - \underline{\theta}$. Here the derivative is $\frac{2}{(y_3 - y_1)}$.

If $y_2 < \theta^* + \underline{\theta}$, the coordinates of B are $\left(0, \frac{y_3 - \theta^* - \underline{\theta}}{y_3 - y_2}, \frac{\theta^* + \underline{\theta} - y_2}{y_3 - y_2}\right)$ and the area below AB has the size $\frac{\sqrt{3}}{2} \cdot \left[1 - \frac{(y_3 - \theta^* - \underline{\theta})^2}{(y_3 - y_2)(y_3 - y_1)}\right]$. Thus, the condition for an interior equilibrium threshold (5) is equivalent to $\theta^* = 1 - \frac{(y_3 - \theta^* - \underline{\theta})^2}{(y_3 - y_2)(y_3 - y_1)}$. The right-hand side is increasing and convex in θ^*. So, the derivative of the right-hand side is maximal at the lowest θ^* at which the condition $y_2 < \theta^* + \underline{\theta}$ applies, *i.e.* at $\theta^* = y_2 - \underline{\theta}$. Here the derivative is also $\frac{2}{(y_3 - y_1)}$.

Combining the two cases, we see that for any interior equilibrium the derivative of the right-hand side of (4) is smaller than 1 if $y_3 - y_1 > 2$.

Thus, we conclude that $y_3 - y_1 > 2$ is a sufficient condition for a unique equilibrium.

QED