Rationalizable Expectations and Sunspot Equilibria in an Overlapping-generations Economy

Introduction

The economy is a social system composed of individual economic actors who are uncertain about each other's behavior. Uncertainty of this sort is called market uncertainty (Shell, 1987, p. 549). In the 1980s two theories for the treatment of market uncertainty have been developed. One is the concept of sunspot equilibria, introduced by Cass and Shell (1983). The other is the concept of rationalizable expectations, developed by Bernheim (1984) and Pearce (1984) as a solution concept for noncooperative games.

McAllister (1988) and Guesnerie (1992) have shown that rationalizable expectations can also be applied to market economies that cannot be written in strategic form. This paper analyses the rationalizable expectations equilibria of an overlapping generations exchange economy. It is shown that rationalizable expectations equilibria are the price and consumption paths that are predicted by the theory of sunspot equilibria if one allows probability beliefs about sunspot activity to differ across individuals. This result allows for a new interpretation of sunspot equilibria and helps to understand their relevance.

Market uncertainty arises whenever the actions of an economic agent depend on his expectations about variables that are or will be influenced by other agents. The simplest paradigm for such a situation is an overlapping generations economy. Demand in one period depends on the individuals' expectations aboutfuture prices. These prices are endogenous variables, depending on the demand of future generations. However, the demand of future generations depends on their expectations about prices in their relative future. Thus, price expectations can be traced back to expectations on expectations. In order to close the model, a solution concept, endogenizing expectations, is needed.

The most popular method of endogenizing expectations is the rational expectations hypothesis (REH). It is based on the idea that individual expectations should be the same as the predictions of the theory (Muth, 1961). A rational expectations equilibrium (REE) is a function, assigning determinate values to all endogenous variables for any possible state of the world. The individuals are assumed to behave as though they know this function. In a model with a single REE, the equilibrium is the prediction of the theory and individual expectations are the same.

Intertemporal allocation models typically have multiple equilibria. In such cases the prediction of the theory is a *set*of equilibria. Beliefs that assign positive probabilities to different equilibria would accord with the theory. But, beliefs of this kind are excluded by the REH. In these cases, the concept of rational expectations does not give a complete description of all economic events consistent with rational behavior. Individuals with rational expectations behave *as if* they are able to predict the behavior of other agents for each state of the world correctly. Thus, instead of dealing with market uncertainty, the REH eliminates it.

Morgenstern (1935) recognized the dilemma inflicted by the hypothesis that individual expectations are

the same as the predictions of the theory: The agents are assumed to behave asthough they know the predictions of a theory which tries to explain theirbehavior. A prediction of the theory requires well defined individual expectations *and vice versa*.

In order to get all equilibria consistent with rationality, Morgenstern (1935) proposed an iterative procedure which successively eliminates all expectations that contradict the predictions of the theory: The first prediction of the theory consists of the events that are consistent with unrestricted expectations. Eliminating expectations that contradict this first prediction leads to a second prediction which is more accurate. In the third round only expectations according with the second prediction are admitted, and so on. The limit of this mental process describes a set of equilibria that are predicted by the theory whatever positive probabilities the agents assign to these equilibria.
Luce and Raiffa (1957) applied this method as iterative elimination of strictly dominated strategies to noncooperative games. Bernheim (1984) and Pearce (1984) strengthened it by considering the independence of individual strategies. They called the strategies surviving the elimination procedure "rationalizable". The term "rationalizable expectations" has been used accordingly for the probability measures supported by rationalizable strategies.

Bernheim (1986), Brandenburger and Dekel (1987), and Tan and Werlang (1988) have analyzed the decision theoretic foundation of rationalizable expectations. They demonstrated that this concept is characterized by assuming that the players of a game are choosing strategies independently of each other in order to maximize their expected utilities while the solution concept is common knowledge. Another assumption, characterizing rationalizable expectations, is that the players have the same information as an outside observer (Heinemann, 1995a).

McAllister (1988) adapted rationalizable expectations to a market economy with asymmetric information (McAllister (1988) calls these expectations "weakly admissible") . Guesnerie (1992) introduced this concept to amacroeconomic model in the spirit of Muth (1961). A general method for the application of decision theoretic solution concepts to market economies that cannot be written in strategic form has been developed by Heinemann (1995a, 1995b). In this paper rationalizable expectations are applied to an overlapping generations exchange economy. To make the arguments as clear as possible, we consider an economy with one commodity per period and one unproductive and useless asset that may be interpreted as fiat money or land.

Sunspot equilibria are another theory for the analysis of marketuncertainty: Aumann (1974) observed that noncooperative games may have equilibria in which the chosen strategies depend on extrinsic random variables whichhave no influence on the pay-off matrix. He named them "a-posteriori equilibria". Cass and Shell (1983) applied this idea to nonstrategic market economies. Cass and Shell designed a non-stochastic economy and introduced a random variable, called "number of sunspots", that is unrelated to the fundamentals of the economy. They showed that there are rational expectations equilibria in which the endogenous variables depend on the number of sunspots. The additional equilibria, obtained by considering extrinsic uncertainty, can be interpreted as being consistent with rational behavior but excluded by the REH. The prediction of this theory is the set of price paths that can arise in an equilibrium

for some sequence of sunspots.

The meaning of "sunspots" is still open to interpretation. They may be viewed as labels for psychological factors (Azariadis and Guesnerie, 1986, p. 725.) or as selecting devices which affect the economy directly, but are excluded fromeconomic analysis ("one may think of political, cultural, and institutional features", Benhabib and Rustichini, 1994, p. 2). This paper suggests that the "number of sunspots" should be interpreted as a label for a price system, like a catalogue number. In a sunspot equilibrium the individuals assign probabilities to numbers which stand for different price systems. This is to avoid modelling subjective probabilities for prices directly. The number of sunspots is not a true exogenous variable, it can only be identified by observing the prices and using the rule of numeration. This interpretation follows naturally from the main result of this paper: The price and consumption paths that are predicted by the rational expectations equilibria of an overlapping generations exchange economy with extrinsic uncertainty are the rational*izable* expectations equilibria of the *non*stochastic economy.

A related result has been provided by Brandenburger and Dekel (1987). In the context of noncooperative game theory they have shown the equivalence of iteratively undominated strategies and Aumann's a-posteriori-equilibria. Forges and Peck (1995) have demonstrated that common belief sunspot equilibria of an overlapping generations economy can be viewed as correlated equilibria of a market game a la Shapley and Shubik (1977). For a wider class of models, Guesnerie (1993a,b) has shown that a sunspot equilibrium is unique if, and only if, there is a unique rationalizable expectations equilibrium. (Without formal proof Guesnerie gives convincing arguments for this result in the framework of an abstract one-step forward looking system.)

The paper is organized as follows: Section 2 introduces the overlapping generations economy. In section 3 the rational expectations equilibria of this economy are discussed. Section 4 applies the concept of rational*izable* expectations. Section 5 states some properties of rationalizable expectations equilibria. Section 6 defines sunspot equilibria for the economy and analyzes their relationship to rationalizable expectations equilibria. Some concluding remarks are given in section 7.