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Force of Friction between Fractal Rough Surface and Elastomer

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Abstract—The force of friction between a fractal rough surface and a model elastomer has been numerically simulated using the method of dimensionality reduction. For elastomers, in which the imaginary part of the complex shear modulus is much greater than the real part, the friction coefficient is proportional to the mean square gradient of the solid surface profile. The empirical value of the proportionality coefficient is close to unity.

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Friction between rough solid surfaces is of considerable importance in many applications. Grosch [1, 2] established that the friction of elastomers is determined by the internal losses in contacting bodies and, hence, is closely related to the rheology of these materials. Simple analytical estimations [3] show that the coefficient of friction between an elastomer and a rough solid surface obeys the following relation:

$$\mu = \xi \nabla z \frac{G''(kv)}{|\hat{G}(kv)|}, \qquad (1)$$

where ∇z is the mean square slope (gradient) of the solid surface profile $z = z(x, y)$, $\hat{G}(\omega)$ is the frequency-dependent complex shear modulus of the elastomer, $G''(\omega)$ is the imaginary part of this complex quantity, k is the characteristic wave vector of the solid surface profile, v is the velocity of mutual sliding, ξ is a dimensionless constant n the order of unity, and $|\hat{G}(\omega)|$ is the modulus of the complex modulus. Since Eq. (1) is derived on the basis of qualitative considerations, the exact value of constant ξ is unknown and can only be determined by means of exact numerical simulations.

Relation (1) acquires a more simple form, provided that the complex shear modulus is purely imaginary (or its imaginary part is much greater than the real part). Then, the ratio $G''(\omega)/|\hat{G}(\omega)|$ is unity and Eq. (1) reduces to

$$\mu = \xi \nabla z. \qquad (2)$$

Since the friction coefficient in this case is independent of the characteristic wave vector of the solid surface profile, it can be suggested that Eq. (2) will also be valid for the surfaces possessing no such a characteristic wave vector, including fractal surfaces. The present investigation was devoted to verifying this hypothesis and numerically calculating the ξ value.

In order to check for the validity of the above assumption, let us consider a simple medium characterized by the purely imaginary shear modulus

$$\hat{G} = i\eta\omega. \qquad (3)$$

which ensures the applicability of Eq. (2). Our numerical simulation employs the method of dimensionality reduction described in [3, 4]. According to this, the three-dimensional (3D) elastomer and the 2D rough surface are modeled by one-dimensional lines. This is achieved by transforming the spectral density of the 2D surface into the spectral density of a line, which retains the invariant mean square values of the height, gradient, and curvature of the surfaces well as their distributions [4]. Below we consider friction in this one-dimensional model, but the results of this investigation are equally valid for the initial 3D system.

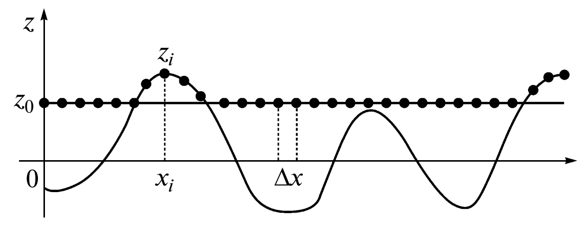

Fig. 1. One-dimensional model used in the method of dimensionality reduction for simulating the contact between an elastomer and a rough solid surface (see text for explanations).

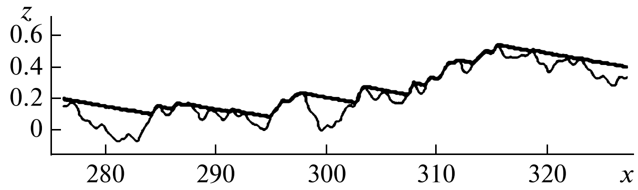

Fig. 2. "Snapshot" of rough solid surface–elastomer contact (thick solid curve) configuration.

The model consists of a fractal rough line (Fig. 1) that is generated as follows:

$$z(x) = Z_0 \int_{q_1}^{q_2} dq c(q) \cos(qx + \zeta), \qquad (4)$$

where $c(q) = q^{-\beta}$ is the spectral density of the one-dimensional rough line; $\beta = H + 1/2$; H is the Hurst parameter; q_1 and q_2 are the characteristic wave vectors of truncation of the spectral density; and $\zeta(x)$ is the random phase, which is assumed to be δ-correlated so that $\langle\zeta(q)\zeta(q')\rangle = 2\pi\delta(q - q')$. In order to accelerate the computational process, new parts of the solid surface are generated in the course of propagation of the elastomer–solid contact region, using an algorithm described in [5]. The elastomer is modeled by a sequence of discrete elements connected to a hard straight line by links with a complex stiffness coefficient $\hat{k} = 4\hat{G}(\omega) = 4i\eta\omega\Delta x$, where Δx is the step of discretization of the one-dimensional system. A complex stiffness coefficient of this type is inherent in shock absorbers, where the force is proportional to the velocity as $F(x_i) = [4\eta\Delta x]\dot{z}(x_i)$. This law ensures the equivalence of the contact properties of the one-dimensional model to those of a real 3D elastomer [3].

The calculations of motions in the system were performed according to the following algorithm. The system is initiated by bringing the solid surface and elastomer into the first contact. From this moment on, a vertical force F_N is acting on the elastomer while the solid counterbody is driven to move in the negative direction of axis x at velocity v. At every step, the contact condition is checked, according to which elements of the elastomer contacting with the solid remain in contact until the interaction force would become negative (we consider the contact without adhesion, for which negative contact forces are prohibited); at this moment, the elastomer–solid contact is entirely lost. In contrast, elements not involved in the contact before, are considered as brought into contact when the distance between the position of an element and the solid surface is zero or negative (in the latter case, the element moves on the surface so that the coordinate difference vanishes). At each temporal step, the normal force is monitored and, if necessary, changed by moving the elastomer as a whole in the vertical direction. In order to determine the tangential force, the normal force for each element is multiplied by the slope of the surface at the given point.

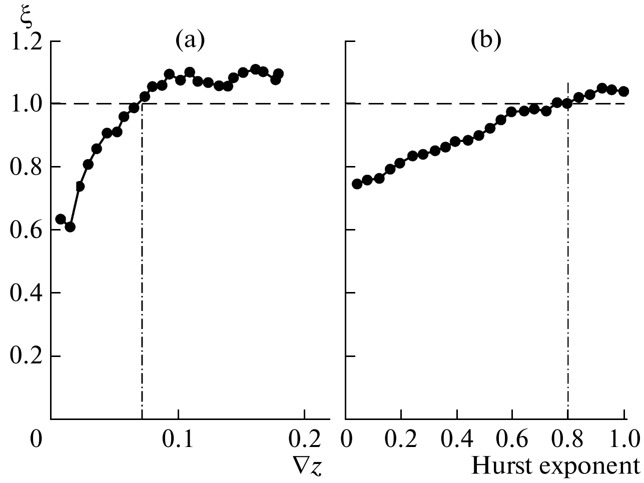

Fig. 3. Plots of coefficient $\xi = \mu/\nabla z$ versus (a) mean square gradient ∇z of the surface profile and (b) Hurst parameter H.

The above procedure yields the surface profile shape that is illustrated for a certain moment of tine in Fig. 2. The origin of this pattern is easy to understand based on the following simple notions. Since the medium consists of "pure shock absorbers," its elements are "pressed upward" by the solid counterbody but do not exhibit back response (except for slow relaxation related to a slow downward motion of the body as a whole). Thus, the elastomer is only pressed to all protrusions of the solid surface from one side, which is just what accounts for the arising friction force.

According to Eq. (2), the ratio of the friction coefficient to the mean square gradient of the surface profile, $\xi = \mu/\nabla z$, must depend neither on the normal force nor on the surface roughness or the characteristic wave vector. Thus, according to out hypothesis, this ratio is independent of the fractal dimension of the solid surface. Figure 3 shows ξ for the surfaces with various properties, in particular, different values of the surface roughness and fractal dimension. As expected for a given force and increasing gradient of the rough surface profile, ξ tends to a constant value (Fig. 3a). This limiting value rather weakly depends on the fractal dimension of the surface (Fig. 3a). For a Hurst parameter of $H = 0.8$, which corresponds to a fractal dimension of $D_f = 2.2$ (that is close to the dimensions of many "natural" surfaces), the ξ value is exactly unity.

Thus, the results of our numerical simulation confirm the validity of analytical estimation given by Eq. (2). The empirical value of coefficient ξ is close to unity.

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