1. Klausur Statik und elementare Festigkeitslehre WS 09/10

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Bitte deutlich in <u>DRUCKSCHRIFT</u> schreiben!

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Name, Vorname:

Matr.-Nr.:

Studiengang:

Bitte links oder rechts ankreuzen!

○ Studienbegleitende Prüfung ○ Übungsscheinklausur

(kg, m, s) ✓

1. Geben Sie die Maßheiten im internationalen Einheitensystem (SI-System) an!

Querkraft im Träger $Q(x)$ N oder $\frac{kg\,m}{s^2}$

Flächenschwerpunkt x_s m

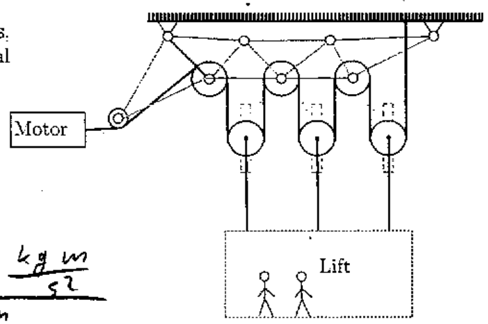
Reaktionslast am Loslager N oder $\frac{kg\,m}{s^2}$

Moment Nm oder $\frac{kg\,m^2}{s^2}$

Druckverteilung $p(x, y)$ $\frac{N}{m^2}$ oder $\frac{kg}{m s^2}$

(2 Punkte)

2. Was ist die Höchstmasse des Liftes, wenn ein elekrischer Motor maximal 2kN aufbringen kann? ($g = 10\text{m/s}^2$)



$6 \cdot 2kN = 12\,kN$

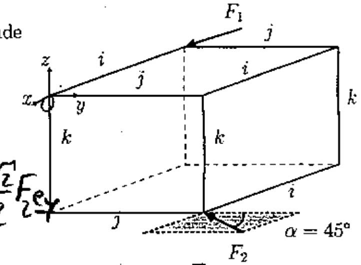
$12\,kN = m \cdot g$

$$m = \frac{12 \cdot 10^3 \frac{kg\,m}{s^2}}{10 \frac{m}{s^2}}$$

$= 1200\,kg$

(2 Punkte)

3. Bestimmen Sie per Kreuzprodukt das resultierende Moment um den Ursprung ,



$$\underline{M} = \sum_{i=1}^{2} \underline{x}_i \times \underline{F}_i = \begin{pmatrix} M_x^{(O)} \\ M_y^{(O)} \\ M_z^{(O)} \end{pmatrix}$$

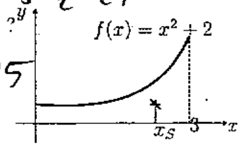
$\underline{F}_1 = F_1 \underline{e}_x$, $\underline{F}_2 = -\frac{\sqrt{2}}{2} F_2 \underline{e}_x - \frac{\sqrt{2}}{2} F_2 \underline{e}_z$

$\underline{x}_1 = -i \underline{e}_x$; $\underline{x}_2 = j \underline{e}_y - k \underline{e}_z$

$$\underline{x}_1 \times \underline{F}_1 = 0 \; ; \; \underline{x}_2 \times \underline{F}_2 = \begin{pmatrix} 0 \\ j \\ -k \end{pmatrix} \times \begin{pmatrix} -\frac{\sqrt{2}}{2} F_2 \\ -\frac{\sqrt{2}}{2} F_2 \\ 0 \end{pmatrix} = \begin{pmatrix} -k \frac{\sqrt{2}}{2} F_2 \\ k \frac{\sqrt{2}}{2} F_2 \\ +j \frac{\sqrt{2}}{2} F_2 \end{pmatrix} = \underline{M}$$

(2 Punkte)

4. Finden Sie den Schwerpunkt x_S für die Füße des Designerstuhles ?



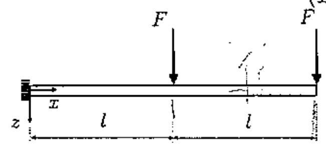
$$\int_0^3 f(x)\,dx = \int_0^3 (x^2+2)\,dx = \frac{x^3}{3} + 2x \Big|_0^3 = 15$$

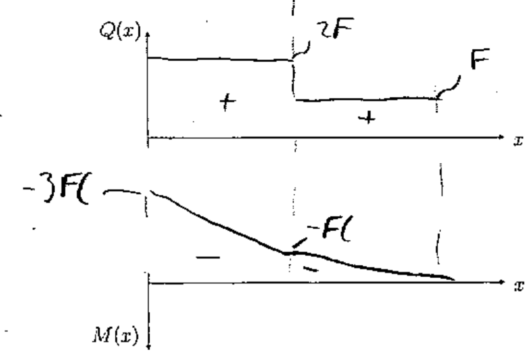
$$\int_0^3 f(x)\,x\,dx = \int_0^3 (x^3+2x)\,dx = \frac{x^4}{4} + x^2 \Big|_0^3 = \frac{81}{4} + 9$$

$$\Rightarrow x_s = \frac{\frac{81}{4} + 9}{15}$$

(1 Punkte)

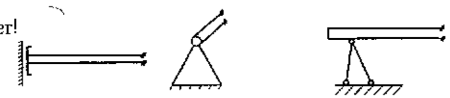
5. Skizzieren Sie qualitativ die Querkraft- und Momentenverläufe für die untengegebene Probe und geben Sie markante Punkte quantitativ an !





(2 Punkte)

6. Bestimmen Sie die Wertigkeiten der Lager!



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(1 Punkte)

1

1

a)

	A_i	x_{si}	y_{si}	$A_i\,x_{si}$	$A_i\,y_{si}$
1	$\frac{a^2\pi}{2}$	a	$6a+\frac{4a}{3\pi}$	$\frac{a^3\pi}{2}$	$3a^3\pi+\frac{2}{3}a^3$
2	$\frac{9}{8}a^2$	a	$\frac{11}{2}a$	$\frac{9}{8}a^3$	$\frac{99}{16}a^3$
3	$3a^2$	$\frac{1}{4}a$	$3a$	$\frac{3}{4}a^3$	$9a^3$
4	$\frac{9}{8}a^2$ (2)	a (2)	$\frac{1}{2}a$ (2)	$\frac{9}{8}a^3$	$\frac{9}{16}a^3$
Σ mit $\pi=3$	$\frac{27}{4}a^2$ (1)			$\frac{9}{2}a^3$ (1)	$\left(\frac{108}{16}+18+\frac{2}{3}\right)a^3$ (1)

b) $x_s = \frac{\sum A_i x_{si}}{\sum A_i} = \frac{18}{27}a$ (1)

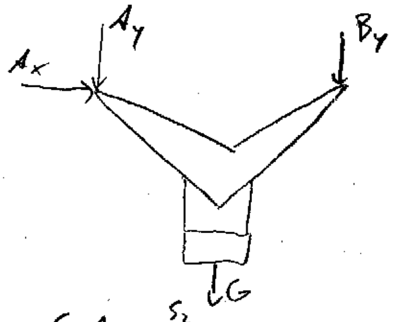
$y_s = \frac{\sum A_i y_{si}}{\sum A_i} = \frac{\left(\frac{108}{16}+18+\frac{2}{3}\right)}{\frac{27}{4}}a$ (1)

c) $S_1 = S_2 = S_3 = S_4$ (2)

Σ (13)

2

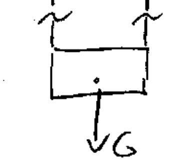
a)



$\sum F_x = 0: \quad A_x = 0$

$\sum M^{(A)} = 0: \quad -G\cdot 4a - B_y\cdot 8a = 0 \Rightarrow B_y = -\frac{G}{2}$ (1)

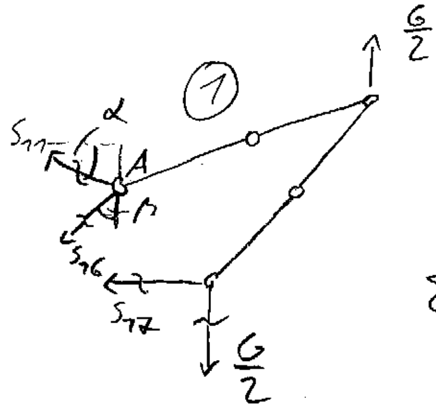
$\sum F_y = 0: \quad -A_y - B_y - G = 0 \Rightarrow A_y = -\frac{G}{2}$



$\sum M^{(1)} = 0: \quad -G\cdot a + S_2\cdot 2a = 0 \Rightarrow S_2 = \frac{G}{2}$

$\sum F_y = 0: \quad S_1 + S_2 - G = 0 \Rightarrow S_1 = \frac{G}{2}$ (1)

b) Nullstäbe: 13, 14, 15, 4, 5, 19, 20, 21 (2) c)



$\beta = 45°$

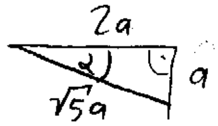
$$\sum M^{(A)} = 0: \quad \frac{G}{2}\cdot 4a - \frac{G}{2}a - S_{17}\, a = 0$$

$$\Rightarrow S_{17} = \frac{3}{2} G \quad (\text{Zug}) \quad 1$$

$$\sum F_x = 0: \quad - S_{17} - S_{16} \cos(45°) - S_{11} \cos(\alpha) = 0 \qquad (1)$$

$\Rightarrow$

α:



1 $\Rightarrow \cos(\alpha) = \frac{2}{\sqrt{5}}$

$\sin(\alpha) = \frac{1}{\sqrt{5}}$

$$\sum F_y = 0: \quad \cancel{\frac{G}{2}} - \cancel{\frac{G}{2}} - S_{16}\cos(45°) + S_{11}\sin(\alpha) = 0$$

$$\Rightarrow \quad S_{16}\cos(45°) = S_{11}\sin(\alpha) \qquad \text{in } (1)$$

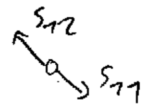
$$-\frac{3}{2}G - S_{11}\sin(\alpha) - S_{11}\cos(\alpha) = 0$$

$$\Rightarrow S_{11} = -\frac{3}{2}G \cdot \frac{\sqrt{5}}{3} = -\frac{\sqrt{5}}{2}G \quad (\text{Druck}) \quad 1$$

$$\Rightarrow S_{16} = -\frac{\sqrt{5}}{2}G\frac{1}{\sqrt{5}}\cdot\frac{2}{\sqrt{2}} = -\frac{G}{\sqrt{2}} \quad (\text{Druck}) \quad 1$$

d)

11:



$S_{12} = S_{11} = -\frac{\sqrt{5}}{2}G$ (Druck) 1

1:

$$\sum F_x = 0: \quad S_{12}\cos(\alpha) + S_1\cos(45°) = 0$$

$$S_1 = -S_{12}\frac{2}{\sqrt{5}}\frac{2}{\sqrt{2}} = G\frac{2}{\sqrt{2}} \quad (\text{Zug}) \quad 1$$

2:

$S_1 = S_2 = G\frac{2}{\sqrt{2}}$ (Zug) 1

aus Symmetrie
oder analoges Freischneiden:

$S_9 = S_{12} = -\frac{\sqrt{5}}{2}G$ (Druck)
$S_8 = S_1 = G\frac{2}{\sqrt{2}}$ (Zug)
$S_{\_} = S_{\_} = G\frac{2}{\_}$ (Zug)

1

$\boxed{3}$

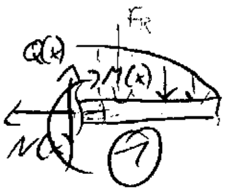
a) $q(x) = A \cos(Bx)$

RBs: $q(x=0) = q_0 \quad \Rightarrow \quad A = q_0$

$q(x=l) = 0 \quad \Rightarrow \quad B = \frac{\pi}{2l}$

$$q(x) = q_0 \cos\left(\frac{\pi}{2l} x\right) \qquad 1$$

b) Variante 1 (Freischnittmethode):



$$\sum F_x = 0: \Rightarrow N(x) = 0 \qquad 1$$

$$\sum F_y = 0: -Q(x) + q_0 \int_x^l \cos\left(\frac{\pi}{2l}\tilde{x}\right) d\tilde{x} = 0 \qquad 1$$

$$Q(x) = q_0 \frac{2l}{\pi} \sin\left(\frac{\pi}{2l}\tilde{x}\right)\Big|_x^l$$

$$= q_0 \frac{2l}{\pi}\left(1 - \sin\left(\frac{\pi}{2l} x\right)\right) \qquad 1$$

Angriffspunkt der Resultierenden:

$$x_R = \frac{q_0 \int_x^l \cos\left(\frac{\pi}{2l}\tilde{x}\right)\tilde{x}\, d\tilde{x}}{\int_x^l \cos\left(\frac{\pi}{2l}\tilde{x}\right) d\tilde{x}}$$

$\Rightarrow$

$$\sum M^{(S)} = 0: \quad -M(x) - F_R \cdot (x_R - x) = 0 \qquad 1$$

$$-F_R x_R = -q_0 \int_x^l \cos\left(\frac{\pi}{2l}\tilde{x}\right)\tilde{x}\, d\tilde{x}$$

$$= -q_0 \frac{2l}{\pi} \sin\left(\frac{\pi}{2l}\tilde{x}\right)\tilde{x}\Big|_x^l + \int_x^l q_0 \frac{2l}{\pi} \sin\left(\frac{\pi}{2l}\tilde{x}\right) d\tilde{x}$$

$$= -\frac{q_0 2l^2}{\pi} + \frac{q_0 2l}{\pi} \sin\left(\frac{\pi}{2l}x\right)x - q_0 \frac{4l^2}{\pi^2} \cos\left(\frac{\pi}{2l}\tilde{x}\right)\Big|_x^l$$

$$= -\frac{q_0 2l^2}{\pi} + \frac{q_0 2l}{\pi} \sin\left(\frac{\pi}{2l}x\right)x + q_0 \frac{4l^2}{\pi^2} \cos\left(\frac{\pi}{2l}x\right) \qquad 1$$

$$\Rightarrow M(x) = -\frac{q_0 2l^2}{\pi} + \cancel{\frac{q_0 2l}{\pi} \sin\left(\frac{\pi}{2l}x\right)x} + q_0 \frac{4l^2}{\pi^2} \cos\left(\frac{\pi}{2l}x\right)$$

$$- \cancel{\frac{q_0 2l}{\pi} \sin\left(\frac{\pi}{2l}x\right)x} + q_0 \frac{2l}{\pi} x$$

$$= -\frac{q_0 2l^2}{\pi} + q_0 \frac{2l}{\pi} x + q_0 \frac{4l^2}{\pi^2} \cos\left(\frac{\pi}{2l}x\right) \qquad 1$$

Variante 2 (Schnittlasten DGL):

$N(x) = 0$ 1

$Q(x)$: $\frac{dQ(x)}{dx} = -q(x)$

$$Q(x) = -\int q(x)\,dx$$
$$= -q_0 \int \cos\left(\frac{\pi}{2l}x\right) dx$$
$$= -q_0 \frac{2l}{\pi} \sin\left(\frac{\pi}{2l}x\right) + C_1 \quad 1$$

RB: $Q(x=l) = 0 \Rightarrow C_1 = q_0 \frac{2l}{\pi}$ 1

$$Q(x) = -q_0 \frac{2l}{\pi} \sin\left(\frac{\pi}{2l}x\right) + \frac{q_0 2l}{\pi}$$
$$= q_0 \frac{2l}{\pi}\left(1 - \sin\left(\frac{\pi}{2l}x\right)\right) \quad 1$$

$M(x)$: $\frac{dM(x)}{dx} = Q(x)$

$$M(x) = \int Q(x)\,dx$$
$$= -\frac{q_0 2l}{\pi} \int \left[\sin\left(\frac{\pi}{2l}x\right) - 1\right] dx$$
$$= -\frac{q_0 2l}{\pi}\left[-\frac{2l}{\pi}\cos\left(\frac{\pi}{2l}x\right) - x\right] + C_2 \quad 1$$

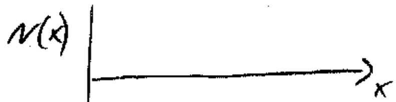
RB: $M(x=l) = 0$

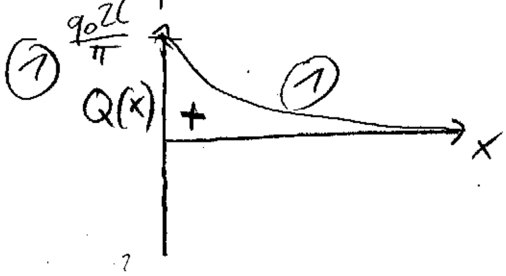
$\Rightarrow C_2 = -\frac{q_0 2l^2}{\pi}$ 1

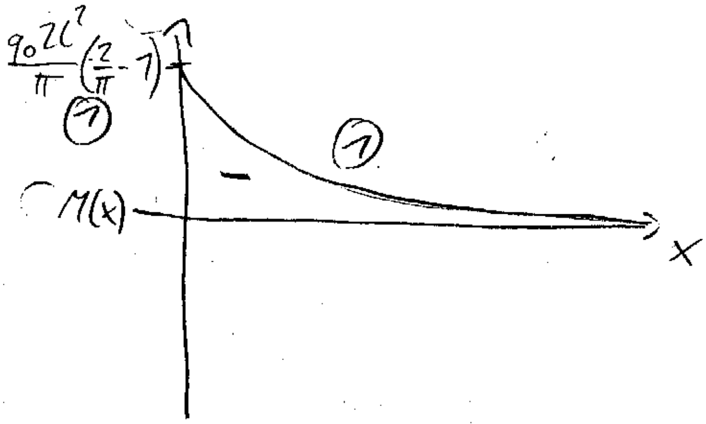
$\Rightarrow$

$$M(x) = -\frac{q_0 2l^2}{\pi} + \frac{q_0 2l}{\pi}x + q_0 \frac{4l^2}{\pi^2}\cos\left(\frac{\pi}{2l}x\right) \quad 1$$

c) Skizzen:







d) $M_{max} = M(x=0) = \frac{q_0\, 2l^2}{\pi}\left(\frac{2}{\pi}-1\right)$