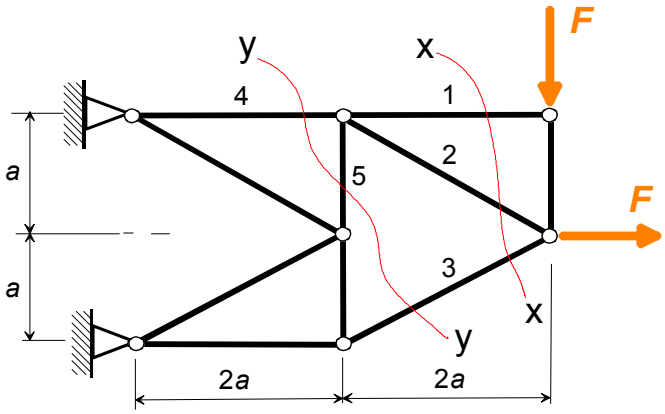
Klausur SS2000 (1)

Aufgabe1

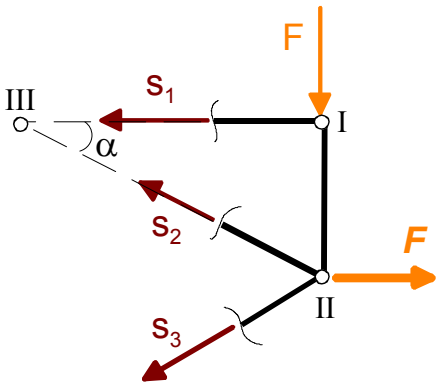


a) $2k = s + r$

$2 \cdot 7 = 10 + 4$

$14 = 14$ $\Rightarrow$ statisch bestimmt

b) Schnitt x-x:



$\sum M_{II} = 0 = s_1 \cdot a$ $\Rightarrow \underline{\underline{s_1 = 0}}$

$\sum M_{III} = 0 = -F \cdot 2a + F \cdot a - s_3 \cdot \cos\alpha \cdot a - s_3 \cdot \sin\alpha \cdot 2a$

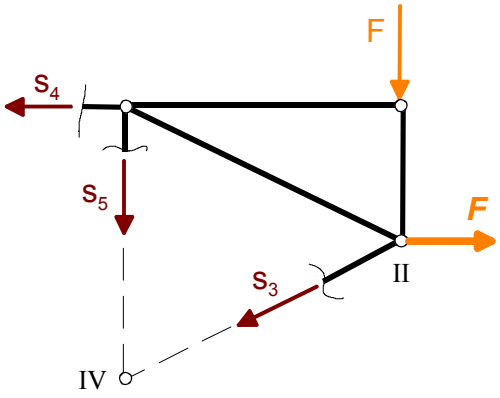
$$\Rightarrow s_3 = -\frac{F}{\cos\alpha + 2 \cdot \sin\alpha}$$

mit $\alpha = \arctan\frac{1}{2} \approx 26{,}6°$ $\Rightarrow \underline{\underline{s_3 = -5{,}59kN}}$ (Druckstab)

$$\sum F_x = 0 = -s_1 - s_2 \cdot \cos\alpha - s_3 \cdot \cos\alpha + F$$

$$\Rightarrow \underline{\underline{s_2}} = \frac{-s_1 - s_3 \cdot \cos\alpha + F}{\cos\alpha} = \frac{5{,}59kN \cdot \cos 26{,}6° + 10kN}{\cos 26{,}6°} = \underline{\underline{16{,}77kN}} \text{ (Zugstab)}$$

Schnitt y-y:

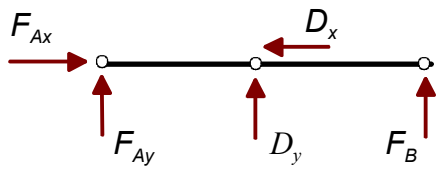


$$\sum M_{IV} = 0 = s_4 \cdot 2a - F \cdot 2a - F \cdot a \qquad \Rightarrow \underline{\underline{s_4}} = \frac{3}{2}F = \underline{\underline{15kN}} \text{ (Zugstab)}$$

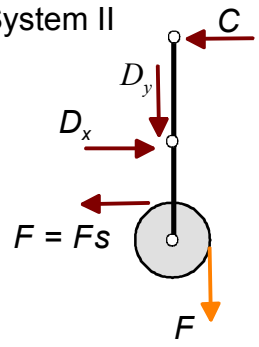
$$\sum M_{II} = 0 = s_4 \cdot a + s_5 \cdot 2a \qquad \Rightarrow \underline{\underline{s_5}} = -\frac{s_4}{2} = \underline{\underline{-7{,}5kN}} \text{ (Druckstab)}$$

Aufgabe2

System I



System II



c) $3 \cdot n = r + v$

$3 \cdot 2 = 3 + 3$

$6 = 6$ $\Rightarrow$ statisch bestimmt

d) System II: $F_s = F$

$\sum M_D = 0 = -F_s \cdot 2a - F \cdot a + F_c \cdot 3a \quad \Rightarrow \underline{\underline{F_C = F}}$

$\sum F_x = 0 = F_{Dx} - F_C - F_S \quad \Rightarrow \underline{\underline{F_{Dx} = 2F}}$

$\sum F_y = 0 = -F_{Dy} - F \quad \Rightarrow \underline{\underline{F_{Dy} = -F}} \quad \Rightarrow \underline{\underline{F_D = \sqrt{F_{Dx}^2 + F_{Dy}^2} = \sqrt{5}F}}$

System I:

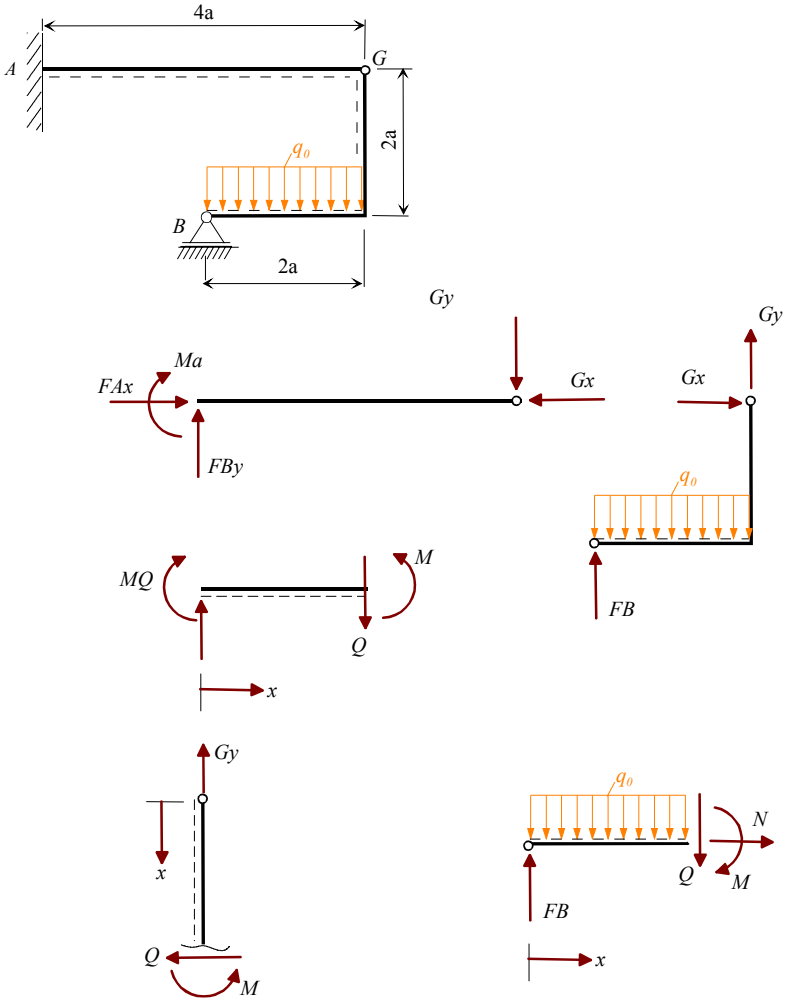
$\sum M_A = 0 = F_B \cdot 8a + F_{Dy} \cdot 4a \quad \Rightarrow \underline{\underline{F_B = -\frac{1}{2} F_{Dy} = \frac{1}{2} F}}$

$\sum F_x = 0 = F_{Ax} - F_{Dx} \quad \Rightarrow F_{Ax} = F_{Dx} = 2F$

$\sum F_y = 0 = F_{Ay} + F_{Dy} + F_B \quad \Rightarrow F_{Ay} = -F_{Dy} - F_A = \frac{1}{2} F$

$\Rightarrow \underline{\underline{F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{4{,}25}F}}$

Aufgabe 3:



a) <u>System II:</u>

$$\sum M_G = 0 = -F_B \cdot 2a + q \cdot 2a^2 \qquad \Rightarrow \underline{\underline{F_B = q_0 \cdot a}}$$

$$\sum F_y = 0 = F_B - q_0 \cdot 2a + G_y \qquad \Rightarrow \underline{\underline{G_y = q_0 \cdot a}}$$

$$\sum F_x = 0 = G_x \qquad \Rightarrow \underline{\underline{G_x = 0}}$$

System I:

$$\sum F_x = 0 = F_{Ax} - G_x \qquad \Rightarrow \underline{\underline{F_{Ax} = 0}}$$

$$\sum F_y = 0 = F_{Ay} - G_y \qquad \Rightarrow \underline{\underline{F_{Ay} = q_0 \cdot a}}$$

$$\sum M_A = 0 = -M_a - G_y \cdot 4a \qquad \Rightarrow \underline{\underline{M_a = -4q_0 \cdot a^2}}$$

b) Bereich $0 \le x \le 4a$:

$$\underline{\underline{N = 0}}$$

$$\underline{\underline{Q = F_{Ay} = q_0 \cdot a}}$$

$$M = M_a + F_{Ay} \cdot x \qquad \Rightarrow \underline{\underline{M_{(0)} = M_a = -4q_0 \cdot a^2}}$$

$$\Rightarrow \underline{\underline{M_{(x=4a)} = 0}}$$

Bereich $0 \le x \le 2a$:

$$\underline{\underline{N = G_y = q_0 \cdot a}}$$

$$\underline{\underline{Q = 0}}$$

$$\underline{\underline{M = 0}}$$

Bereich $0 \le x \le 2a$:

$$\underline{\underline{N = 0}}$$

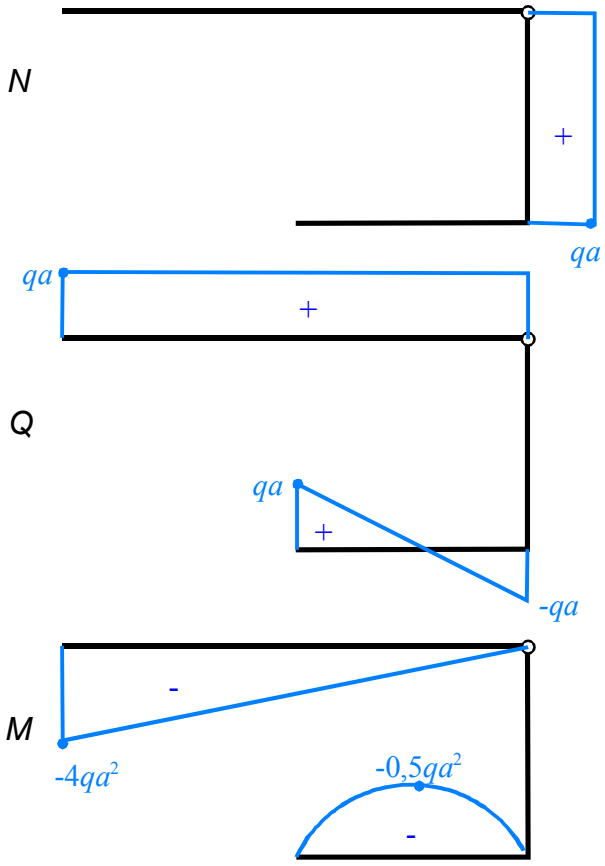
$$Q = F_B - q_0 \cdot x = q_0 (a - x) \qquad \Rightarrow \underline{\underline{Q_{(x=0)} = q_0 \cdot a}}$$

$$\Rightarrow \underline{\underline{Q_{(x=2a)} = -q_0 \cdot a}}$$

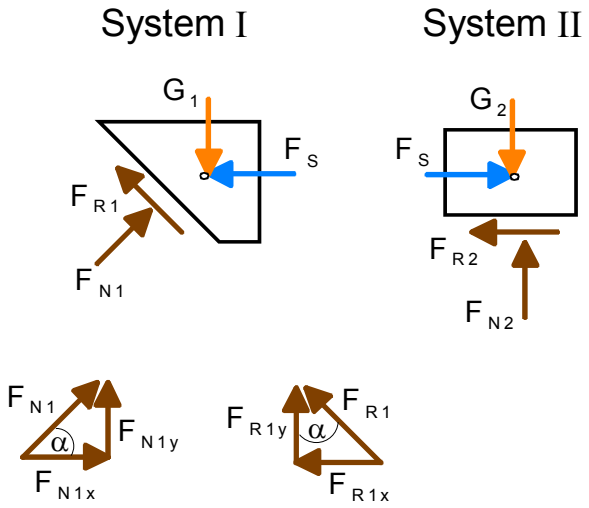
$$M = -F_B \cdot x + q_0 \cdot \frac{x^2}{2} = q_0 \left(\frac{x^2}{2} - a \cdot x \right) \qquad \Rightarrow \underline{\underline{M_{(x=0)} = 0}}$$

$$\Rightarrow \underline{\underline{M_{(x=0)} = -\frac{1}{2} q_0 \cdot a^2}}$$

$$\Rightarrow \underline{\underline{M_{(x=2a)} = 0}}$$


N
+
qa
qa
+
Q
qa
+
-qa
-
M
-4qa²
-0,5qa²
-

Aufgabe 4:



$$F_{R1} = \mu_1 \cdot F_{N1} \qquad (1)$$

$$F_{R2} = \mu_2 \cdot F_{N2} \qquad (2)$$

System I:

$$\sum F_x = 0 = F_{N1} \cdot \cos\alpha - F_{R1} \cdot \sin\alpha - F_S = F_{N1} \cdot \cos\alpha - \mu_1 \cdot F_{N1} \cdot \sin\alpha - F_S$$

$$\Rightarrow F_S = F_{N1}\left(\cos\alpha - \mu_1 \cdot \sin\alpha\right) \qquad (3)$$

$$\sum F_y = 0 = F_{N1} \cdot \sin\alpha + F_{R1} \cdot \cos\alpha - G_1 = F_{N1} \cdot \sin\alpha + \mu_1 \cdot F_{N1} \cdot \cos\alpha - G_1$$

$$\Rightarrow F_{N1} = \frac{G_1}{\sin\alpha + \mu_1 \cdot \cos\alpha} \qquad (4)$$

(4) in (3)

$$\Rightarrow F_S = \frac{G_1\left(\cos\alpha - \mu_1 \cdot \sin\alpha\right)}{\sin\alpha + \mu_1 \cdot \cos\alpha} \qquad (5)$$

System 2:

$$\sum F_x = 0 = F_S - F_{R2} \qquad \Rightarrow F_S = F_{R2} = \mu_2 \cdot F_{N2} \qquad \Rightarrow F_{N2} = \frac{F_S}{\mu_2} \qquad (6)$$

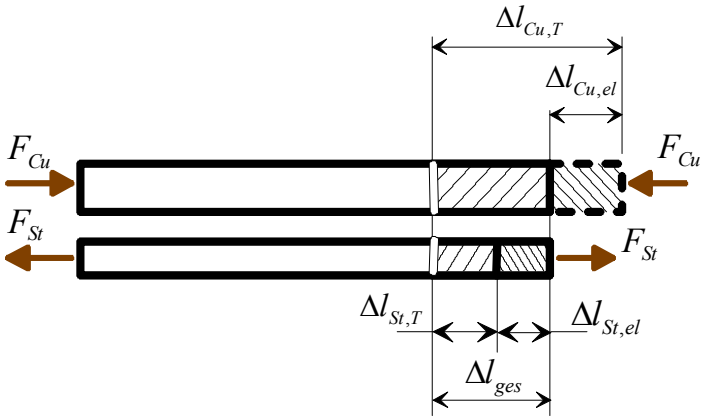
$$\sum F_y = 0 = -G_2 + F_{N2} \qquad \Rightarrow G_2 = F_{N2} = \frac{F_S}{\mu_2} = \frac{G_1(\cos\alpha - \mu_1 \cdot \sin\alpha)}{\mu_2(\sin\alpha + \mu_1 \cdot \cos\alpha)}$$

$$\Rightarrow \underline{\underline{G_2}} = \frac{500N \cdot (\cos 30° - 0{,}3 \cdot \sin 30°)}{0{,}4 \cdot (\sin 30° + 0{,}3 \cdot \cos 30°)} \underline{\underline{= 1178N}}$$

$\underline{\underline{F_{N1} = 658N}}$

$\underline{\underline{F_S = 471{,}5N}}$

Aufgabe 5:



e) $F_{Cu} = F_{St} = F$

$$\Delta l_{Cu_{ges}} = \Delta l_{St_{ges}}$$

$$\Delta l_{T_{Cu}} - \Delta l_{\sigma_{Cu}} = \Delta l_{T_{St}} + \Delta l_{\sigma_{St}}$$

$$\alpha_{Cu} \cdot l \cdot \Delta T - \frac{F_{Cu} \cdot l}{E_{Cu} \cdot A_{Cu}} = \alpha_{St} \cdot l \cdot \Delta T + \frac{F_{St} \cdot l}{E_{St} \cdot A_{St}}$$

$$(\alpha_{Cu} - \alpha_{St})\Delta T = F\left(\frac{1}{E_{Cu} \cdot A_{Cu}} + \frac{1}{E_{St} \cdot A_{St}}\right)$$

$$F = \frac{(\alpha_{Cu} - \alpha_{St})\Delta T}{\frac{1}{E_{Cu} \cdot A_{Cu}} + \frac{1}{E_{St} \cdot A_{St}}}$$

$$F = \frac{\left(17\cdot10^{-6} - 11\cdot10^{-6}\right)K^{-1}\cdot 50^{\circ}C}{\frac{1}{1{,}05\cdot10^{5}\cdot1473N} + \frac{1}{2{,}1\cdot10^{5}\cdot491N}} = \frac{3\cdot10^{-4}}{1{,}61683\cdot10^{-8}}N$$

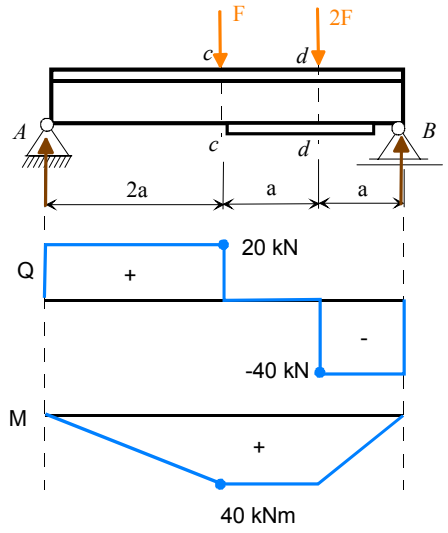
$$F = 1{,}855\cdot10^{4}\,N$$

$$\sigma_{Cu} = \frac{-F}{A_{Cu}} = -12{,}6\frac{N}{mm^2}$$

$$\sigma_{St} = \frac{F}{A_{St}} = 38\frac{N}{mm^2}$$

f) $\tau_{Niet} = \frac{F}{2\cdot A_{Niet}} = 29{,}5\frac{N}{mm^2}$

Aufgabe 6



$$\sum M_A = 0 = F\cdot 2m + 2F\cdot 3m - F_B\cdot 4m$$
$$\rightarrow F_B = 2F = 40kN$$
$$\sum F_y = 0 \rightarrow F_A = 3F - F_B = F = 20kN$$
$$M_c = F_A\cdot 2m = 40kNm$$
$$M_d = F_B\cdot 1m = 40kNm$$

Schnitt C-C:

	A [cm²]	y [cm]	A·y [cm³]	a = |y_i - y_s| [cm]	a^2·A [cm⁴]	I_o [cm⁴]
260·10	26	0,5	13	10,74	2999	-
360·10	36	19	684	7,76	2167,83	3888
∑	62		697			

$$y_c = \frac{697cm^4}{62cm^2} = 11{,}24cm$$

$I_{ges} = 9054{,}83\ cm^4$

$$\boxed{\sigma = \frac{M_C}{I} \cdot e}$$

$$\sigma_1 = \frac{-4000Ncm}{9054{,}83cm^4} \cdot 11{,}24cm = -4{,}96\frac{KN}{cm^2} \approx -50\frac{N}{mm^2} \qquad \text{(Druck)}$$

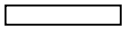
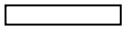
$$\sigma_2 = \frac{-4000kNcm}{9054{,}83cm^4} \cdot 10{,}24cm = -4{,}52\frac{kN}{cm^2} = -45\frac{N}{mm^2} \qquad \text{(Druck)}$$

$$\sigma_3 = \frac{+4000kNcm}{9054{,}83cm^4} \cdot 25{,}76cm = 11{,}38\frac{kN}{cm^2} = 113{,}8\frac{N}{mm^2} \qquad \text{(Zug)}$$

Schweißnaht: $\boxed{\tau = \frac{Q_C \cdot St}{I \cdot 2a}}$

$$\tau_2 = \frac{20kN \cdot 26cm^2 \cdot 10{,}74cm}{9054{,}83cm^4 \cdot 2 \cdot 0{,}4cm} = 0{,}77\frac{kN}{cm^2} = 7{,}7\frac{N}{mm^2}$$

Schnitt d-d:

	A cm^2	y cm	A·y cm^3	a = $\lvert y_i - y_s \rvert$ cm^4	$a^2 \cdot A$	I_o cm^4
 260·10	26	0,5	13	20,97	11433,26	-
 360·10	36	19	684	2,47	219,63	3888
 269·15	39	37,75	1472,25	16,28	10336,5	-
$\sum$	101	-	2151,25	-	21989,39	3888

$$y_d = \frac{2151{,}25cm^3}{101cm^2} = 21{,}47cm$$

$$I = 21989{,}39 + 3888 = 25877{,}39cm^4$$

$$\boxed{\sigma = \frac{M_d}{I} \cdot e}$$

$$\sigma_1 = \frac{-4000\text{kNcm}}{25877{,}39\text{cm}^4} \cdot 21{,}47\text{cm} = -3{,}318\frac{\text{kN}}{\text{cm}^2} = -33{,}18\frac{\text{N}}{\text{mm}^2}$$

$$\sigma_2 = \frac{-4000\text{kNcm}}{25877{,}39\text{cm}^4} \cdot 20{,}47\text{cm} = -3{,}16\frac{\text{kN}}{\text{cm}^2} = -31{,}6\frac{\text{N}}{\text{mm}^2}$$

$$\sigma_3 = \frac{4000\text{kNcm}}{25877{,}39\text{cm}^4} \cdot 15{,}53\text{cm} = 2{,}4\frac{\text{kN}}{\text{cm}^2} = 24\frac{\text{N}}{\text{mm}^2}$$

$$\sigma_4 = \frac{4000\text{kNcm}}{25877{,}39\text{cm}^4} \cdot 17{,}03\text{cm} = 26{,}3\frac{\text{N}}{\text{mm}^2}$$

Schweißnaht: $\boxed{\tau = \frac{Q_d \cdot St}{I \cdot 2a}}$

$$\tau_2 = \frac{40\text{kN} \cdot 26\text{cm}^2 \cdot 20{,}97\text{cm}}{25877{,}39\text{cm}^4 \cdot 2 \cdot 0{,}4\text{cm}} = 1{,}05\frac{\text{kN}}{\text{cm}^2} = 10{,}5\frac{\text{N}}{\text{mm}^2}$$

$$\tau_3 = \frac{40\text{kN} \cdot 39\text{cm}^2 \cdot 16{,}28\text{cm}}{25877{,}39\text{cm}^4 \cdot 2 \cdot 0{,}4\text{cm}} = 1{,}226\frac{\text{kN}}{\text{cm}^2} = 12{,}26\frac{\text{N}}{\text{mm}^2}$$

Aufgabe 7

$$M_A = -F \cdot 2\text{m} = -40\ \text{Nm}$$
$$F_A = F = 20\ \text{N}$$
$$M_i = -F \cdot 1\text{m} = -20\ \text{Nm}$$

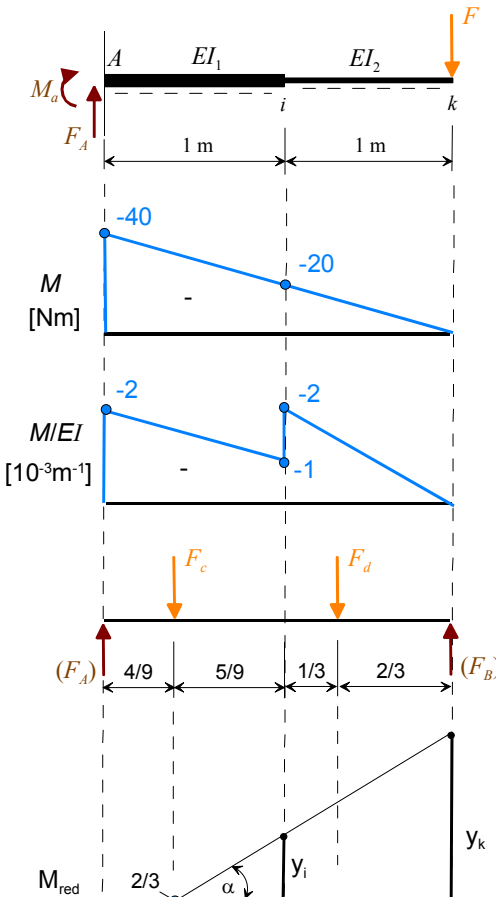
Reduzierte Momente:

$$\frac{M_A}{EI_1} = \frac{-40\ \text{Nm}}{20 \cdot 10^3\ \text{Nm}^2} = -2 \cdot 10^{-3}\ \text{m}^{-1}$$

$$\frac{M_i}{EI_1} = \frac{-20\ \text{Nm}}{20 \cdot 10^3\ \text{Nm}^2} = -1 \cdot 10^{-3}\ \text{m}^{-1}$$

$$\frac{M_i}{EI_2} = \frac{-20\ \text{Nm}}{10 \cdot 10^3\ \text{Nm}^2} = -2 \cdot 10^{-3}\ \text{m}^{-1}$$

Schwerpunkt: Fläche:



$$S = \frac{a+2b}{3(a+b)} \cdot \ell \qquad F = \frac{a+b}{2} \cdot \ell$$

$$S = \frac{2+2\cdot 1}{3(2+1)} \cdot 1\,\mathrm{m} = \frac{4}{9}\,\mathrm{m}$$

$$F_c = \frac{-(2+1)\cdot 10^{-3}\ \mathrm{m}^{-1}}{2} \cdot 1\mathrm{m} = -1{,}5\cdot 10^{-3}$$

$$F_d = -\frac{1}{2} \cdot 1\mathrm{m} \cdot 2\cdot 10^{-3}\ \mathrm{m}^{-1} = -1\cdot 10^{-3}$$

$$\sum M_A = 0 = (F_B)\cdot 2\mathrm{m} - |F_d| \cdot \frac{4}{3}\mathrm{m} - |F_C| \cdot \frac{4}{9}\mathrm{m}$$

$$\rightarrow (F_B) = |F_d| \cdot \frac{2}{3} - |F_C| \cdot \frac{2}{9} = 1\cdot 10^{-3}$$

$$(F_A) = |F_C| + |F_d| - (F_B) = 1{,}5\cdot 10^{-3}$$

Ersatzmomente:

$$M_C = (F_A) \cdot \frac{4}{9}\mathrm{m} = 1{,}5\cdot 10^{-3} \cdot \frac{4}{9}\mathrm{m} = 0{,}\bar{6}\cdot 10^{-3}\ \mathrm{m}$$

$$M_d = (F_B) \cdot \frac{2}{3}\mathrm{m} = 0{,}\bar{6}\cdot 10^{-3}\mathrm{m}$$

$$M_i = (F_B) \cdot 1\mathrm{m} - F_d \cdot \frac{1}{3}\mathrm{m} = 1\cdot 10^{-3}\cdot 1\mathrm{m} - 1\cdot 10^{-3} \cdot \frac{1}{3}\mathrm{m} = 0{,}\bar{6}\cdot 10^{-3}\mathrm{m}$$

$$\alpha = \arctan\left(\frac{0{,}\bar{6}\cdot 10^{-3}}{0{,}\bar{4}}\right) = 0{,}086^\circ$$

$$y_i = \frac{5}{9}\mathrm{m} \cdot \tan\alpha = 0{,}833\cdot 10^{-3}\mathrm{m}$$

$$y_k = 2\mathrm{m} \cdot \tan\alpha = 3\cdot 10^{-3}\mathrm{m}$$