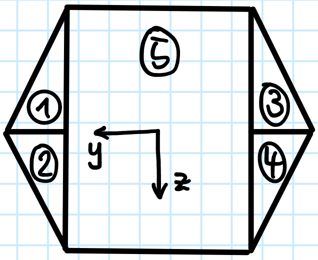
Schriftlicher Test – Statik und elementare Festigkeitslehre, WiSe 19/20

1a)

Bereichseinteilung:
(oder ähnlich)



(1 Punkt)

Für die Dreiecke gilt wegen des Hinweises:

$$I_{yy,1} = I_{yy,2} = I_{yy,3} = I_{yy,4} = \frac{1}{12}\left(\frac{a}{2}\right)\left(\frac{\sqrt{3}}{2}a\right)^3$$

$$= \frac{1}{12}\,\frac{a}{2}\,\frac{3\sqrt{3}}{8}\,a^3 = \underline{\underline{\frac{\sqrt{3}}{64}a^4}} \quad \text{(1 Punkt)}$$

Für das Rechteck gilt:

$$I_{yy,5} = \frac{1}{12}a\left(\sqrt{3}a\right)^3 = \frac{3\sqrt{3}}{12}a^4 = \underline{\underline{\frac{\sqrt{3}}{4}a^4}} \quad \text{(1 Punkt)}$$

Gesamtes Profil:

$$I_{yy} = \sum_{i=1}^{5} I_{yy,i} = 4\frac{\sqrt{3}}{64}a^4 + \frac{\sqrt{3}}{4}a^4 = \left(\frac{1}{16}+\frac{1}{4}\right)\sqrt{3}\,a^4$$

$$= \underline{\underline{\frac{5}{16}\sqrt{3}\,a^4}} \quad \text{(1 Punkt)}$$

b)

$$\frac{dN}{dx} = -n(x) = 0 \Rightarrow N(x) = C_N$$

Randbedingung: $N(x=\ell) = F \Rightarrow \underline{\underline{N(x) = F}}$ (1 Punkt)

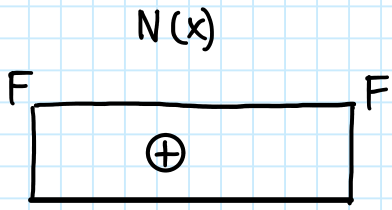
$$\frac{dQ}{dx} = -q(x) = -q_0 \Rightarrow \frac{dM}{dx} = Q(x) = q_0\ell\left(C_Q - \left(\frac{x}{\ell}\right)\right)$$

$$\Rightarrow M(x) = q_0\ell^2\left(C_Q\left(\frac{x}{\ell}\right) - \frac{1}{2}\left(\frac{x}{\ell}\right)^2 + C_M\right) \quad \text{(1 Punkt)}$$

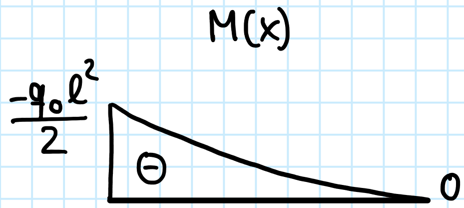
Randbedingungen:

$$\begin{cases} Q(x=\ell) = 0 \\ M(x=\ell) = 0 \end{cases} \Rightarrow \begin{cases} C_Q = 1 \\ C_M = -\frac{1}{2} \end{cases} \quad \text{(1 Punkt)}$$

$$\underline{\underline{M(x) = q_0\ell^2\left(-\frac{1}{2}\left(\frac{x}{\ell}\right)^2 + \left(\frac{x}{\ell}\right) - \frac{1}{2}\right)}} \quad \text{(1 Punkt)}$$



(1 Punkt)



(1 Punkt)

c)

$$\sigma(x,z) = \frac{N(x)}{A} + \frac{M(x)}{I_{yy}} z \qquad \text{(1 Punkt)}$$

$$A = \sum_{i=1}^{5} A_i = 4\left(\frac{1}{2}\,\frac{a}{2}\,\frac{\sqrt{3}}{2}a\right) + a\sqrt{3}a = \frac{\sqrt{3}}{2}a^2 + \sqrt{3}a^2$$

$$= \underline{\underline{\frac{3}{2}\sqrt{3}\,a^2}} \qquad \text{(1 Punkt)}$$

$$I_{yy} = \frac{5}{16}\sqrt{3}\,a^4$$

$$\sigma(x,z) = \frac{2F}{3\sqrt{3}a^2} + \frac{16}{5\sqrt{3}a^4}\left(-\frac{1}{2}\left(\frac{x}{\ell}\right)^2 + \left(\frac{x}{\ell}\right) - \frac{1}{2}\right) \underline{\underline{q_0 \ell^2 z}} \qquad \text{(1 Punkt)}$$

d)

Betragsmäßig größte Spannung liegt bei $z = -\frac{\sqrt{3}}{2}a$ vor, da

$M(x) < 0$ und $\frac{N(x)}{A} > 0$.

$$\sigma\left(x, z = -\frac{\sqrt{3}}{2}a\right) = \frac{2F}{3\sqrt{3}a^2} - \frac{16}{10}\,\frac{q_0 \ell^2}{a^3}\left[-\frac{1}{2}\left(\frac{x}{\ell}\right)^2 + \left(\frac{x}{\ell}\right) - \frac{1}{2}\right] \qquad \text{(1 Punkt)}$$

Das Maximum tritt bei $x = 0$ auf, da $M(x)$ dort maximal ist.

$$\sigma_{max} = \sigma\left(x=0, z=-\frac{\sqrt{3}}{2}a\right) = \frac{2F}{3\sqrt{3}a^2} - \frac{16}{10}\,\frac{q_0\ell^2}{a^3}\left[-\frac{1}{2}\right]$$

$(|\sigma_{max}| = \ldots)$

$$= \underline{\underline{\frac{2}{3\sqrt{3}}\,\frac{F}{a^2} + \frac{4}{5}\,\frac{q_0 \ell^2}{a^3}}} \qquad \text{(1 Punkt)}$$

2a)

$$EI_{yy}\, w^{IV}(x) = q(x) = q_0\left(5\left(\frac{x}{\ell}\right) - 1\right)$$

$$EI_{yy}\, w'''(x) = q_0 \ell\left(\frac{5}{2}\left(\frac{x}{\ell}\right)^2 - \left(\frac{x}{\ell}\right) + C_1\right) \qquad \text{(1 Punkt)}$$

$$EI_{yy}\, w''(x) = q_0 \ell^2\left(\frac{5}{6}\left(\frac{x}{\ell}\right)^3 - \frac{1}{2}\left(\frac{x}{\ell}\right)^2 + C_1\left(\frac{x}{\ell}\right) + C_2\right) \qquad \text{(1 Punkt)}$$

$$EI_{yy}\, w'''(x) = q_0 \ell \left(\frac{5}{2} \left(\frac{x}{\ell}\right) - \left(\frac{x}{\ell}\right) + C_1 \right) \qquad (1 \text{ Punkt})$$

$$EI_{yy}\, w''(x) = q_0 \ell^2 \left(\frac{5}{6} \left(\frac{x}{\ell}\right)^3 - \frac{1}{2}\left(\frac{x}{\ell}\right)^2 + C_1\left(\frac{x}{\ell}\right) + C_2 \right) \qquad (1 \text{ Punkt})$$

$$EI_{yy}\, w'(x) = q_0 \ell^3 \left(\frac{5}{24} \left(\frac{x}{\ell}\right)^4 - \frac{1}{6}\left(\frac{x}{\ell}\right)^3 + \frac{C_1}{2}\left(\frac{x}{\ell}\right)^2 + C_2\left(\frac{x}{\ell}\right) + C_3 \right) \qquad (1 \text{ Punkt})$$

$$EI_{yy}\, w(x) = q_0 \ell^4 \left(\frac{1}{24} \left(\frac{x}{\ell}\right)^5 - \frac{1}{24}\left(\frac{x}{\ell}\right)^4 + \frac{C_1}{6}\left(\frac{x}{\ell}\right)^3 + \frac{C_2}{2}\left(\frac{x}{\ell}\right)^2 + C_3\left(\frac{x}{\ell}\right) + C_4 \right) \qquad (1 \text{ Punkt})$$

b)

$$\left.\begin{array}{l} w(x=0)=0 \\ w'(x=0)=0 \end{array}\right\} \quad (1 \text{ Punkt})$$

$$\left.\begin{array}{l} w(x=\ell)=0 \\ M(x=\ell)=-EI_{yy}w''(x=\ell)=0 \end{array}\right\} \quad (1 \text{ Punkt})$$

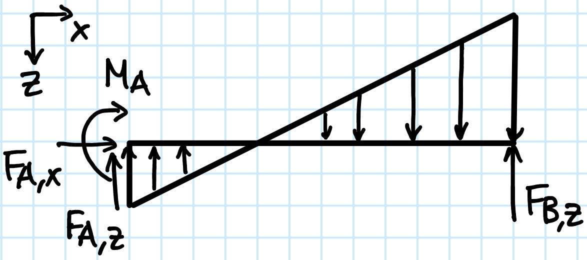
c)

$$\left\{\begin{array}{l} w(x=0)=0 \\ w'(x=0)=0 \\ w(x=\ell)=0 \\ M(x=\ell)=0 \end{array}\right. \iff \left\{\begin{array}{ll} \underline{\underline{C_4=0}} & (1 \text{ Punkt}) \\ \underline{\underline{C_3=0}} & (1 \text{ Punkt}) \\ -\frac{1}{24}+\frac{1}{24}+\frac{C_1}{6}+\frac{C_2}{2}+C_3+C_4=0 & \\ -\frac{1}{2}+\frac{5}{6}+C_1+C_2=0 & \end{array}\right.$$

$$\iff \left\{\begin{array}{l} C_3=C_4=0 \\ C_1=-3C_2 \\ -3C_2+C_2=\frac{1}{2}-\frac{5}{6}=-\frac{1}{3} \end{array}\right. \iff \left\{\begin{array}{ll} C_3=C_4=0 & \\ C_1=-3C_2=\underline{\underline{-\frac{1}{2}}} & (1 \text{ Punkt}) \\ \underline{\underline{C_2=\frac{1}{6}}} & (1 \text{ Punkt}) \end{array}\right.$$

$$\underline{\underline{w(x)=\frac{q_0\ell^4}{EI_{yy}}\left[\frac{1}{24}\left(\frac{x}{\ell}\right)^5-\frac{1}{24}\left(\frac{x}{\ell}\right)^4-\frac{1}{12}\left(\frac{x}{\ell}\right)^3+\frac{1}{12}\left(\frac{x}{\ell}\right)^2\right]}} \quad (1 \text{ Punkt})$$

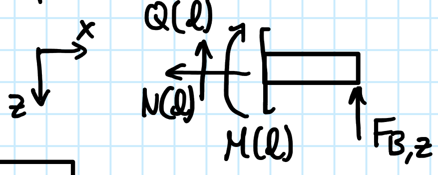
d) Freischnitt:



$\boxed{\text{ODER}}$

(1 Punkt)

infinitesimaler Schnitt bei $x=\ell$:



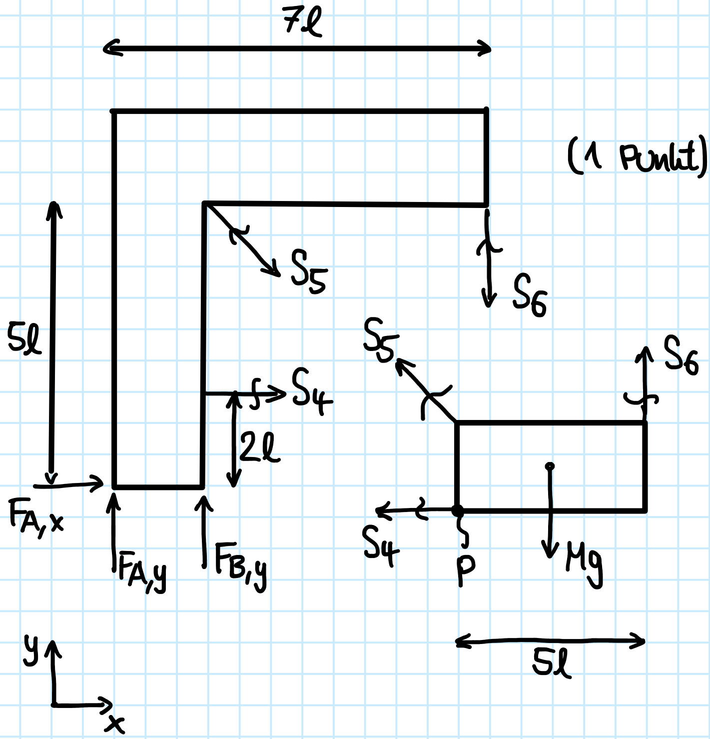
$$\sum F_z \overset{!}{=} 0: \ -Q(\ell)-F_{B,z}=0$$

$$F_{B,z}=\underline{\underline{-Q(\ell)}} \quad (1 \text{ Punkt})$$

$$F_{B,z}=EI_{yy}w'''(x=\ell)=q_0\ell\left(-1+\frac{5}{2}-\frac{1}{2}\right)=\left(\frac{-2+5-1}{2}\right)q_0\ell$$

$$=\underline{\underline{q_0\ell}} \quad (1 \text{ Punkt})$$

3a) Freischnitt:



(1 Punkt)

(1 Punkt)

b)

Gleichgewichtsbedingungen für die Scheibe:

$$\begin{cases} \Sigma F_y = \frac{\sqrt{2}}{2} S_5 + S_6 - Mg \overset{!}{=} 0 & (1) \\ \Sigma F_x = -\frac{\sqrt{2}}{2} S_5 - S_4 \overset{!}{=} 0 & (2) \\ \Sigma M^{(P)} = -Mg \frac{5}{2} l + S_5 \frac{\sqrt{2}}{2} 2l + S_6 5l \overset{!}{=} 0 & (3) \end{cases}$$

(1 Punkt)

aus (3):

$$S_5 = \frac{1}{\sqrt{2}} \left[\frac{5}{2} Mg - 5 S_6 \right] \qquad (4)$$

in (1): $\frac{5}{4} Mg - \frac{5}{2} S_6 + S_6 - Mg = 0 \iff -\frac{3}{2} S_6 = -\frac{1}{4} Mg$

$$\Rightarrow \quad S_6 = \frac{Mg}{6}$$ (1 Punkt)

in (4):

$$S_5 = \frac{1}{\sqrt{2}} \left[\frac{5}{2} - \frac{5}{6} \right] Mg = \frac{5}{3\sqrt{2}} Mg$$ (1 Punkt)

in (2):

$$S_4 = -\frac{\sqrt{2}}{2} S_5 = -\frac{5}{6} Mg$$ (1 Punkt)

Gleichgewichtsbedingungen für das Fachwerk:

$$\begin{cases} \Sigma F_x = F_{A,x} + \frac{\sqrt{2}}{2} S_5 + S_4 \stackrel{!}{=} 0 & (5) \\ \Sigma F_y = F_{A,y} + F_{B,y} - \frac{\sqrt{2}}{2} S_5 - S_6 \stackrel{!}{=} 0 & (6) \qquad (1 \text{ Punkt}) \\ \Sigma M^{(A)} = F_{B,y}\, l - S_4\, 2l - \frac{\sqrt{2}}{2} S_5\, 5l - \frac{\sqrt{2}}{2} S_5\, l - S_6\, 7l \stackrel{!}{=} 0 & (7) \end{cases}$$

aus (7):

$$F_{B,y} = 2 S_4 + 3\sqrt{2}\, S_5 + 7 S_6 = -\frac{5}{3} Mg + 5 Mg + \frac{7}{6} Mg$$

$$= \left(-\frac{10}{6} + \frac{30}{6} + \frac{7}{6}\right) Mg = \frac{27}{6} Mg = \underline{\underline{\frac{9}{2} Mg}} \qquad (1 \text{ Punkt})$$

aus (6):

$$F_{A,y} = -F_{B,y} + \frac{\sqrt{2}}{2} S_5 + S_6 = \left(-\frac{27}{6} + \frac{5}{6} + \frac{1}{6}\right) Mg$$

$$= -\frac{21}{6} Mg = \underline{\underline{-\frac{7}{2} Mg}} \qquad (1 \text{ Punkt})$$

aus (5):

$$F_{A,x} = -\frac{\sqrt{2}}{2} S_5 - S_4 = -\frac{5}{6} Mg + \frac{5}{6} Mg = \underline{\underline{0}} \qquad (1 \text{ Punkt})$$

c)

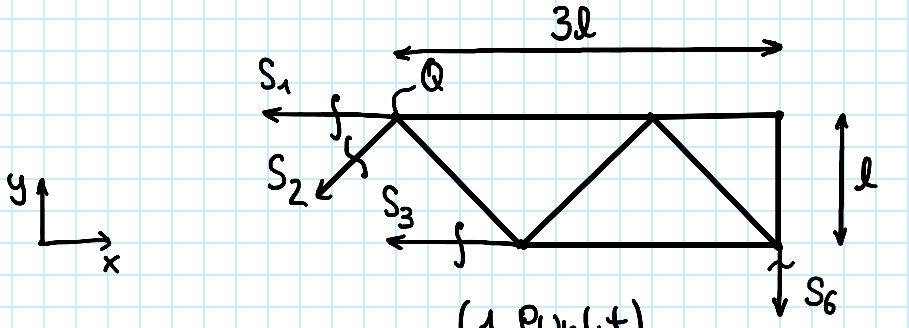
$2k = s + r \qquad k = 19,\ s = 35,\ r = 3$

$s + r = 38,\ 2k = 38 \implies 2k = s + r \qquad (1 \text{ Punkt})$

Damit ist das Fachwerk statisch bestimmt.

d)

Ritterscher Schnitt durch die Stäbe 1, 2, 3



(1 Punkt)

Gleichgewichtsbedingungen:

$$
\begin{cases}
\sum F_x = -S_1 - S_3 - \frac{\sqrt{2}}{2} S_2 \overset{!}{=} 0 \\
\sum F_y = -\frac{\sqrt{2}}{2} S_2 - S_6 \overset{!}{=} 0 \\
\sum M^{(Q)} = -S_3 l - S_6 3l \overset{!}{=} 0
\end{cases}
$$

(1 Punkt)

$$S_3 = -3S_6 = -\frac{Mg}{2}$$ (1 Punkt)

$$S_2 = -\frac{2}{\sqrt{2}} S_6 = -\frac{\sqrt{2}}{6} Mg = -\frac{1}{3\sqrt{2}} Mg$$ (1 Punkt)

$$S_1 = -S_3 - \frac{\sqrt{2}}{2} S_2 = \frac{Mg}{2} + \frac{1}{6} Mg = \frac{2}{3} Mg$$ (1 Punkt)