Scaling of shallow water models

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  - Governing equations
  - Computational examples

- **Anisotropic porosity-based method**
  - Governing equations
  - Computational examples

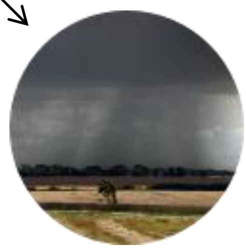
- **Conclusions**

Introduction

Introduction

Processes modeled with shallow water equations

Flooding
(classical)



Rainfall-runoff
(new)

High-resolution data,
efficient survey methods
+
Robust shallow water models

High resolution:
HPC (Distributed
Memory/GPU)

Coarse resolution:
coarse grid methods

- friction law based
- porosity based
- …

Predictions: fast, real time

All images taken from
http://wikipedia.org

Introduction

- Shallow water flow processes are influenced at different scales and a **large bandwith of scales** must be considered, e.g. in **urban flooding** from pavement edges (~ 10cm) over buildings (~ 10m) to the whole city (~ 10 km) or in a **catchment hydrology** from microtopogaphy (local depressions, ~ 1m) over a hillslope (~ 1km) to the whole catchment (~ 100km).

- One possibility to take all scales into account, is to **resolve** them all leading to **high-resolution grids** with very small cell sizes.

- Kim *et al.* (2014) show that the **computational cost** *C* for Godunov based flow solvers is related to the **cell size** *L* as:

$$C \approx k / L^3$$

where *k* is a factor dependent on the numerical scheme.

Introduction

- **Coarse grid methods** solve the shallow water equations on a coarse mesh and take the unresolved influences conceptually into account.
- This significantly reduces the computational time, thus allows fast simulations with acceptable accuracy.

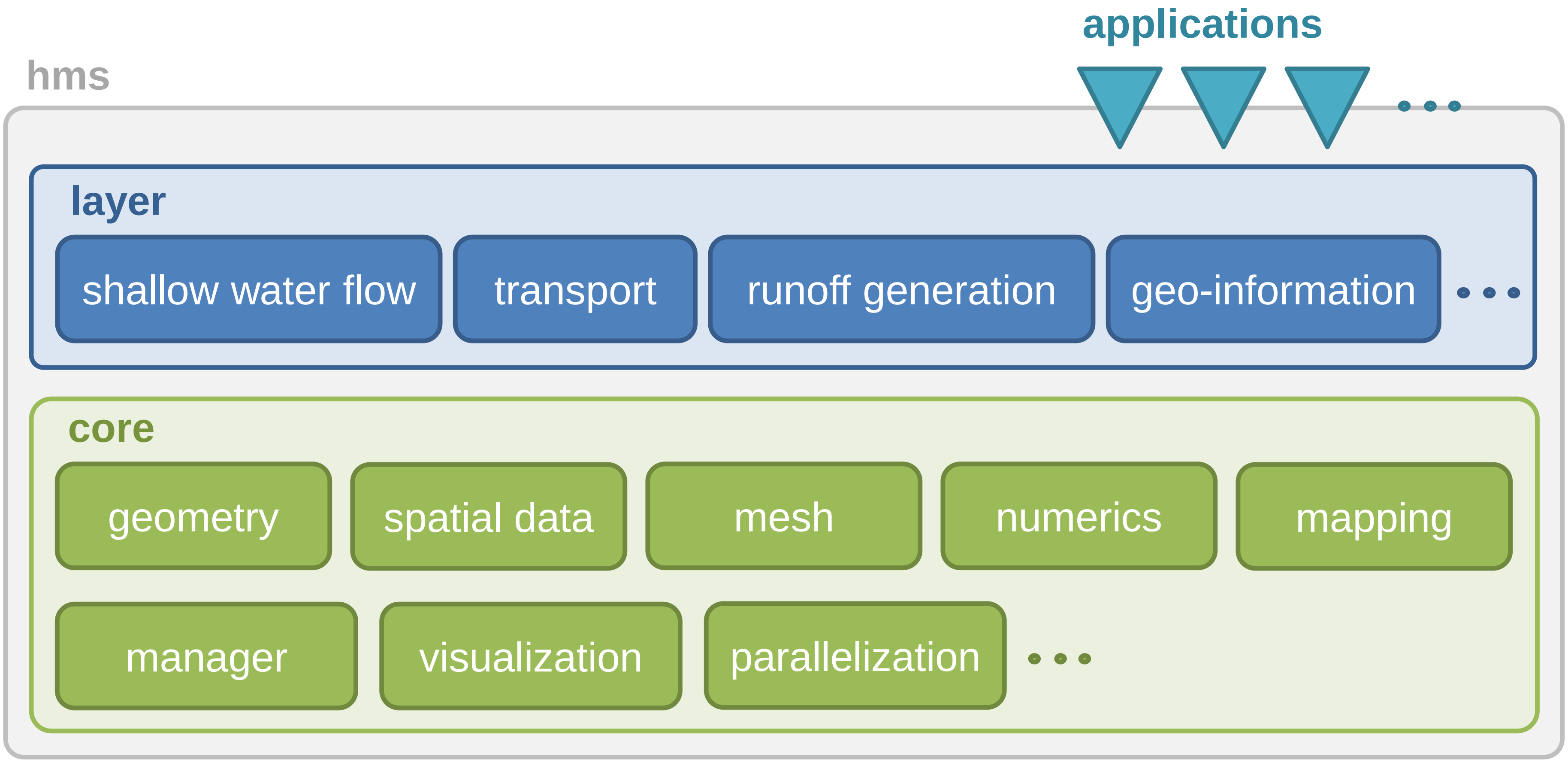
- In the last two years, two coarse grid methods for shallow water flow have been developed at the Chair of Water Resources Management and Modeling of Hydrosystems (TU Berlin):
    1. **Friction law-based** upscaling approach developed by Özgen *et al.* (2015b, 2015c) and Teuber (2015)
    2. **Anisotropic porosity-based** upscaling approach developed by Özgen *et al.* (2015a, nd.a, nd.b)

- The approaches have been implemented in the **Hydroinformatics Modeling System** (hms).

Hydroinformatics Modeling System

- hms is a Java-based object-oriented modeling framework which solves shallow water flow and associated processes using a cell-centered Finite-Volume Method (Simons *et al.* 2014).
- ‘Easy‘ implementation of extensions, e.g. new conceptual approaches, coupling of processes
- ‘Easy‘ handling of spatial data
- Developed at the Chair of Water Resources Management and Modeling of Hydrosystems, TU Berlin, Germany

Software design



Busse *et al.* (2012), Simons *et al.* (2014)

Shallow water equations

- The **two-dimensional shallow water equations** are a special case of the Navier-Stokes equations and can be written as:

$$\frac{\partial \vec{q}}{\partial t} + \frac{\partial \vec{f}}{\partial x} + \frac{\partial \vec{g}}{\partial y} = \vec{s}$$

$$\vec{q} = \begin{bmatrix} h \\ q_x \\ q_y \end{bmatrix}, \quad \vec{f} = \begin{bmatrix} q_x \\ v_x q_x + 0.5gh^2 \\ v_x q_y \end{bmatrix}, \quad \vec{g} = \begin{bmatrix} q_y \\ v_y q_x \\ v_y q_y + 0.5gh^2 \end{bmatrix}$$

where $\vec{q}$ denotes the vector of conserved variables, $\vec{f}$ and $\vec{g}$ stand for the flux vectors in x- and y-direction, respectively, and s is the source term vector.

Shallow water equations

- The **source term** usually consists of mass source, bed slope source and friction slope source term:

$$\vec{s} = \begin{bmatrix} -s_m \\ s_{b,x} + s_{f,x} \\ s_{b,y} + s_{f,y} \end{bmatrix}$$

Here, s_m is the mass source term which can account for rainall, $s_{b,x}$ and $s_{b,y}$ are the bottom slope source terms in x- and y-direction, respectively, and $s_{f,x}$ and $s_{f,y}$ are the friction slope source terms in x- and y-direction, respectively.

Shallow water equations

- The **bottom slope source terms** can be calculated as:

$$\vec{s}_{b,x} = ghS_{0,x}$$

$$\vec{s}_{b,z} = ghS_{0,y}$$

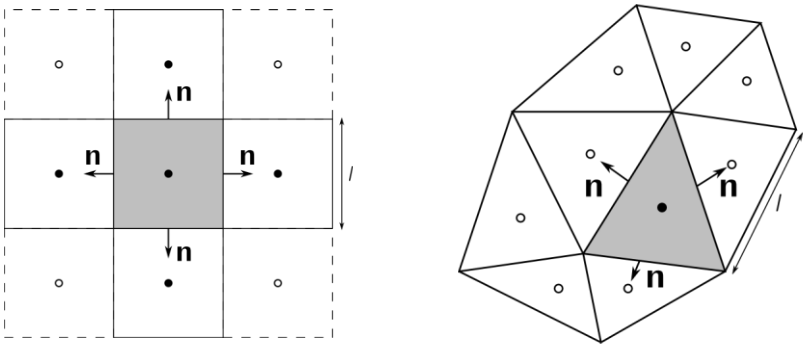
whereby $S_{0,x}$ and $S_{0,y}$ are the bottom slopes in x- and y-direction, respectively, cf. Guinot (2003).

- The **friction slope source term** can be generalized as:

$$\vec{s}_f = Kh^{\alpha} \left| \vec{u} \right|^{\beta} \vec{u}$$

where $\vec{s}_f$ is the vector of the friction term, K is a coefficient, α and β are (in theory) arbitrary parameters (e.g. α=-1/3 and β=1 gives Manning's friction law), cf. (Smith 2014).

Godunov-type scheme



- General formulation of **cell-centered Finite-Volume method**:

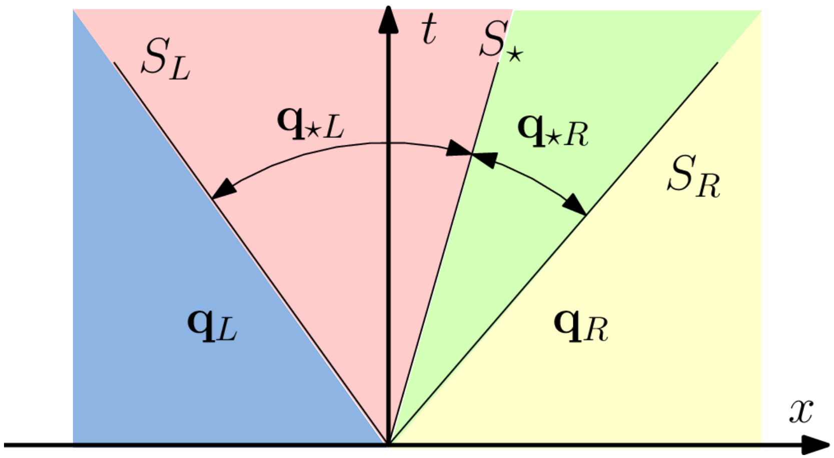
$$\vec{q}^{\,n+1} = \vec{q}^{\,n} - \frac{\Delta t}{A}\sum_k \vec{F}_k^n \vec{n}_k l_k + \Delta t \vec{s}^{\,n}$$

n is the time level, $\vec{n}_k$ is the normal vector of edge k, $\vec{F}$ is the flux vector over the edge k, calculated as $\vec{F}_k = \vec{f}_k \vec{n}_{k,x} + \vec{g}_k \vec{n}_{k,y}$ and l_k is the length of the edge.

- **Godunov-type schemes** calculate $\vec{F}$ by evaluating the **Riemann problem** at the edge.

Godunov-type scheme

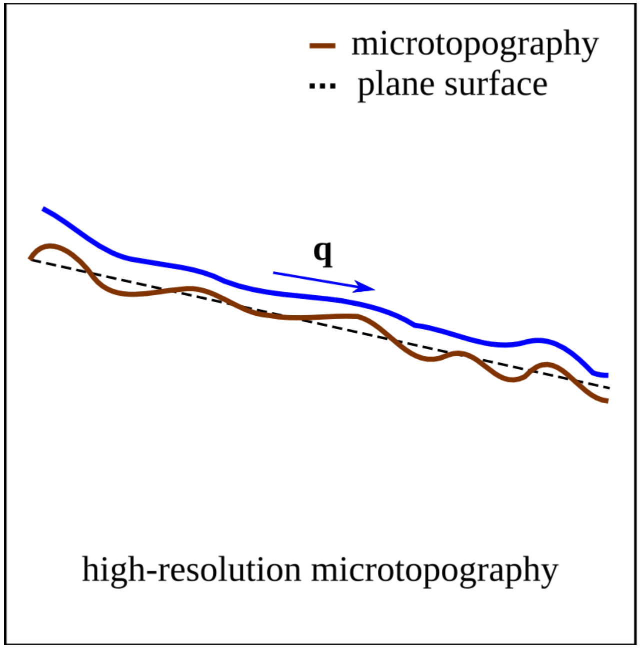
- In this work, an **HLLC Riemann solver** is used to approximate the solution of the Riemann problem.
- The general structure of the HLLC Riemann solution is as follows:

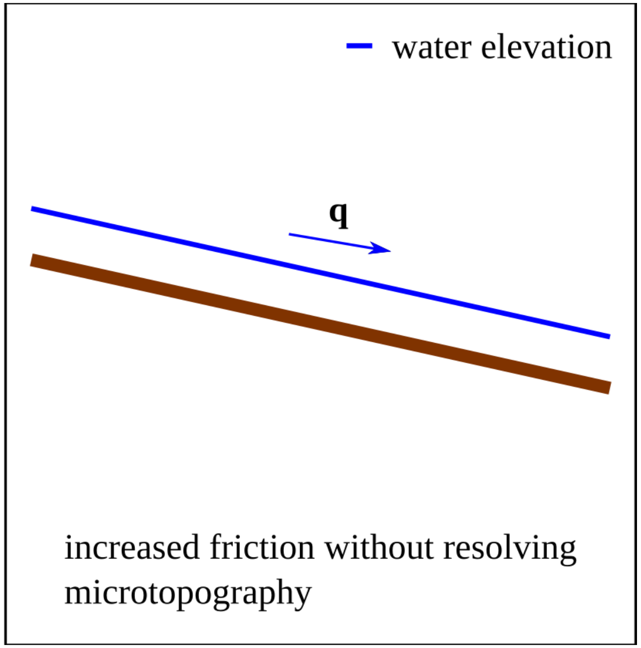


- It allows **efficient solution** of SWE and any number of other processes which are not influencing the Riemann solution directly.
- It is **second order accuracy in space**, spurious oscillations in the solution are avoided by using **TVD methods** developed by Hou *et al.* (2013).

Friction law-based upscaling method

Friction law-based upscaling method





Friction law-based upscaling method

- Friction law-based upscaling methods account for unresolved structures by **increasing the roughness** of the model.
  - Very simple implementation (+)
  - Intuitive approach because it complies with the definition of roughness (+)
  - Not very sophisticated (-)
  - May require extensive model calibration (-)

Governing equations

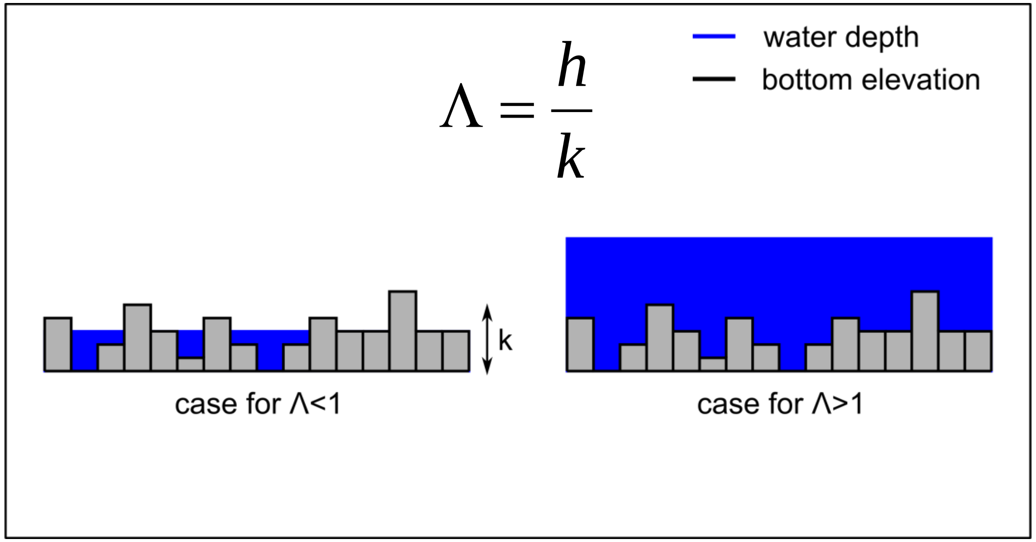
- Many traditional friction laws relate the friction to the water depth *h.*

- The developed upscaling approach uses a dimensionless variable Λ, called **inundation ratio**, instead of the water depth.

- The inundation ratio was initially introduced by Lawrence (1997).

Governing equations

- The **inundation ratio** Λ expresses the relationship between the water depth *h* and the roughness height *k.*



- In this work, the **standard deviation of the microtopography** is taken as the roughness height *k*.

Özgen *et al.* (2015b)

Governing equations

- **The friction slope source term** is calculated as follows (**RM**):

$$\vec{s}_f = \left(\frac{g}{C^2} + K \right) |\vec{u}| \vec{u}$$

Calibration parameter #3

Calibration parameter #1

$$K = \beta_0 \exp\left(-\beta_1 (\Lambda - 1) \right)$$

Calibration parameter #2

$$\boxed{\Lambda = \frac{h}{(1 - I)k}}$$

(Modified inundation ratio to account for the influence of slope)

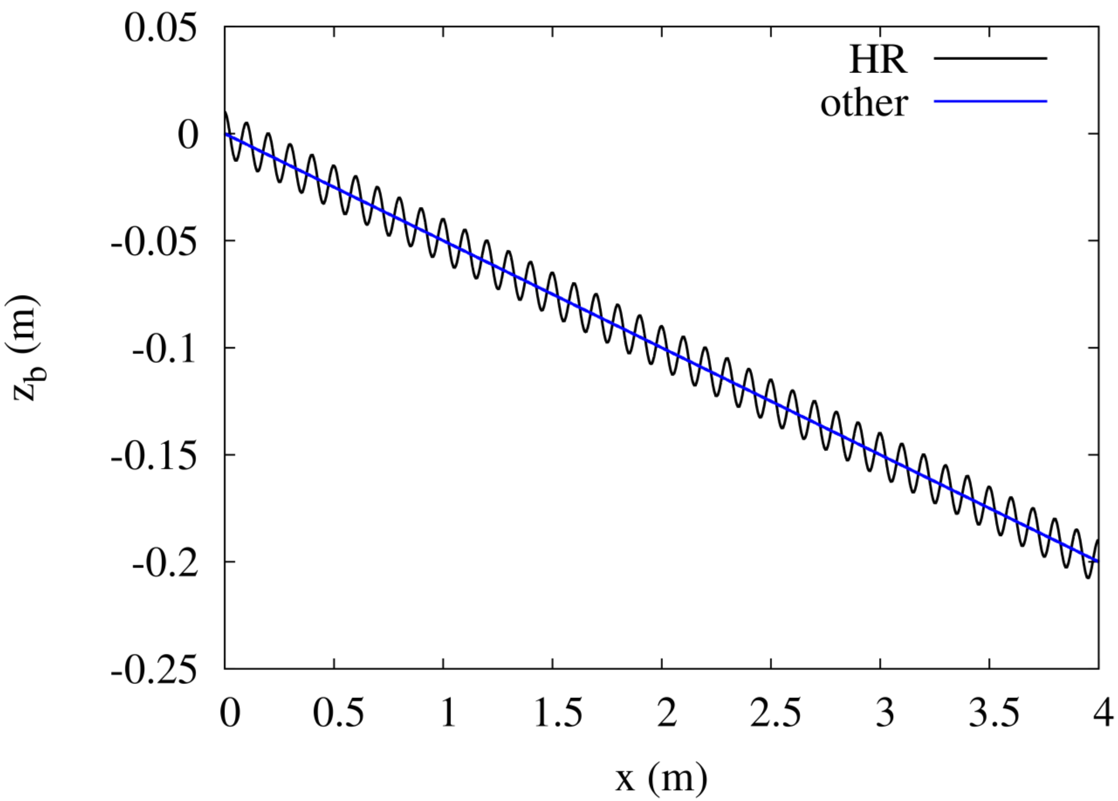
C is the Chezy coefficient, β_0 is a dimensionless friction coefficient and β_1 denotes a geometric conveyance parameter and *I* is the slope.

- The friction law is calibrated with **3 parameters**.

Rainfall-runoff over an inclined plane with sine-wave shaped microtopography (1)

- Constant rainfall is applied to an inclined 4 m long plane with sine-wave shaped microtopography: This has become one of the **standard benchmarks for coarse grid methods** because in the limit every method has to converge to this simple one-dimensional case.

Rainfall-runoff over an inclined plane with sine-wave shaped microtopography (1)

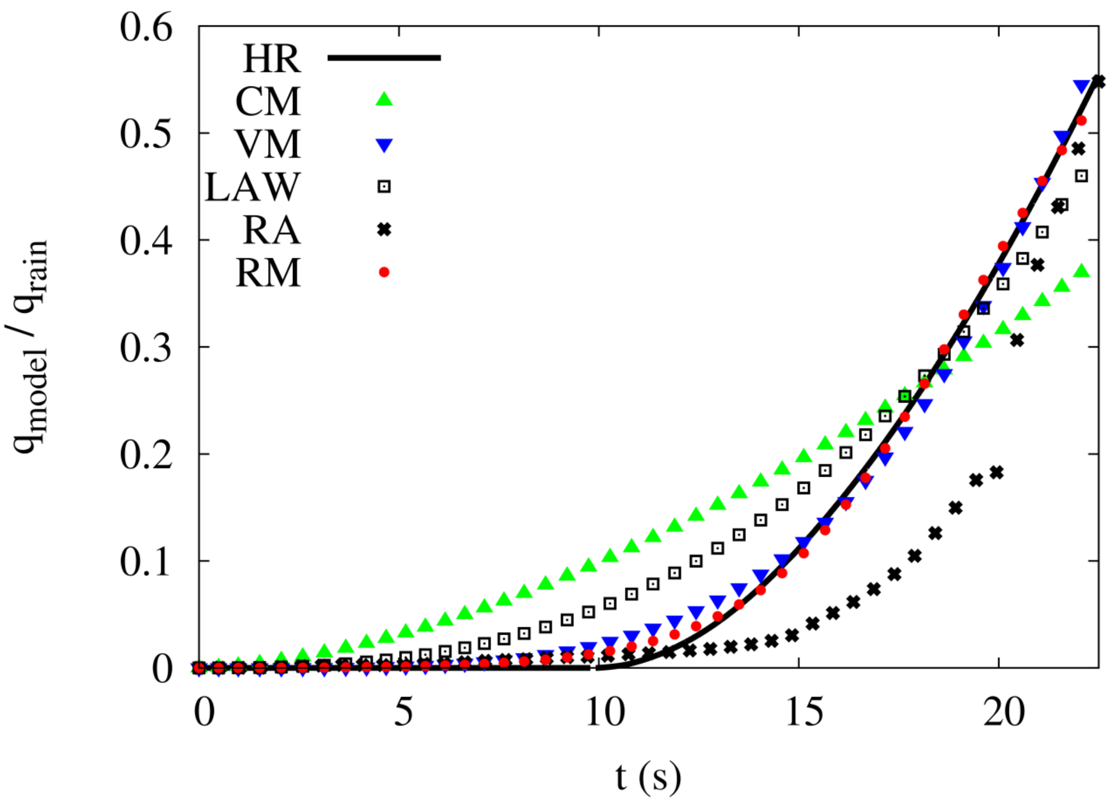


- The standard deviation of the microtopography is σ = 0.01m.
- The HR model uses cells with length Δx = 0.01m.
- All other models use cells with length Δx = 0.1m.
- Rain intensity is $i = 8 \cdot 10^{-4}$ m/s.

Özgen *et al.* (2015b)

Rainfall-runoff over an inclined plane with sine-wave shaped microtopography (1)

- Results show that the proposed **roughness model (RM)** and the **variable Manning's approach (VM)** agree best with the high-resolution simulation (HR) (RMSE(RM) = 0.007, RMSE(VM) = 0.014).



Özgen *et al.* (2015b)

Rainfall-runoff over an inclined plane with sine-wave shaped microtopography (1)

- The HR model has 400 cells, the coarse grid models have 40 cells.
- The **speedup** of all upscaled models is about the same, the simulations run around **58 times faster** than the high-resolution model.

Rainfall-runoff in a small alpine catchment (2)

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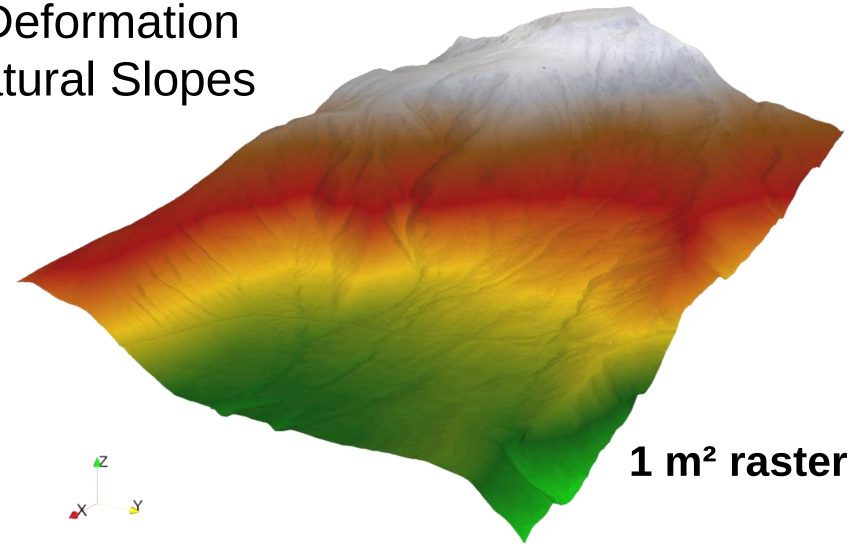
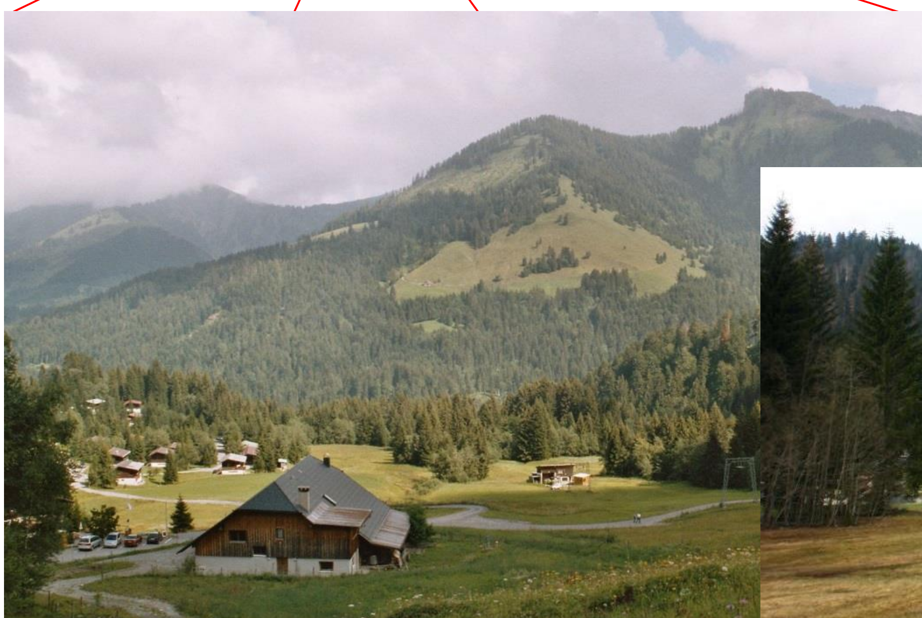
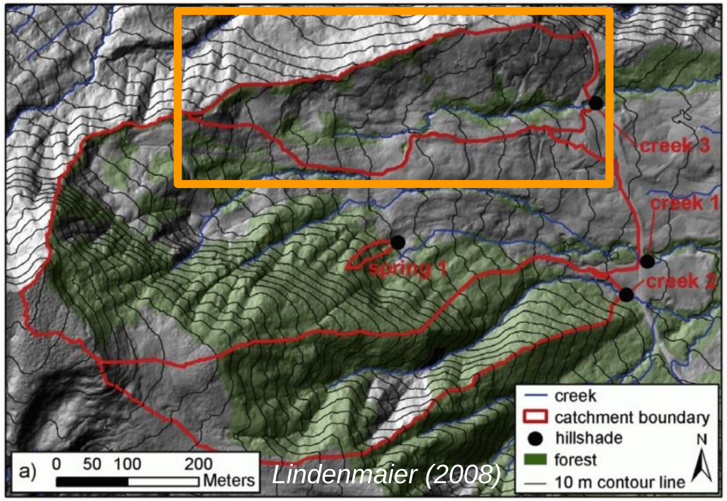
www.grosshang.de

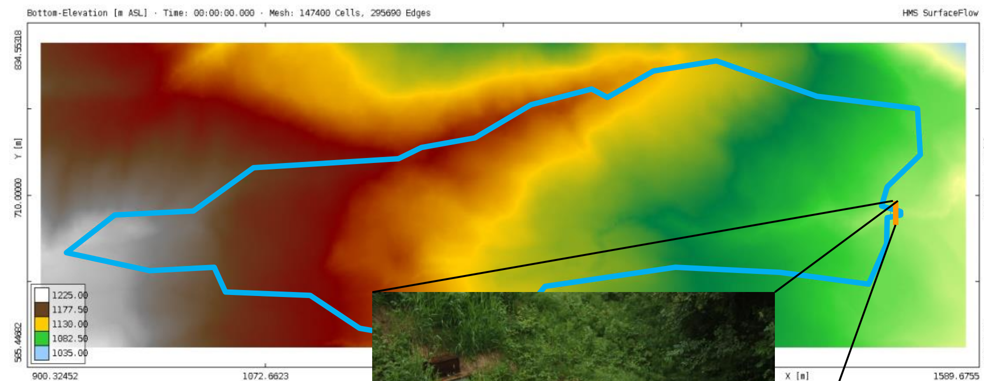


1 m² raster

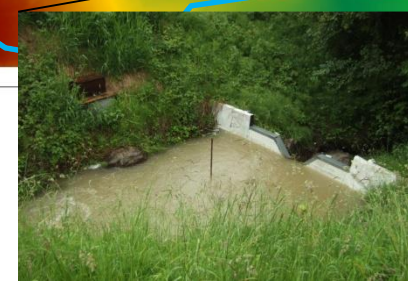


Rainfall-runoff in a small alpine catchment (2): Preliminary studies



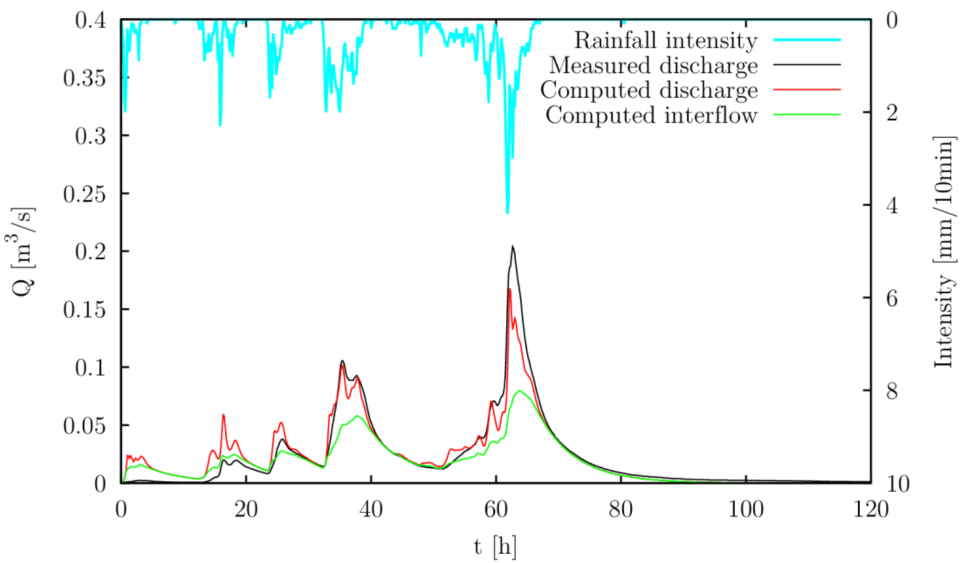


- 1m² DTM
- 147,400 cells

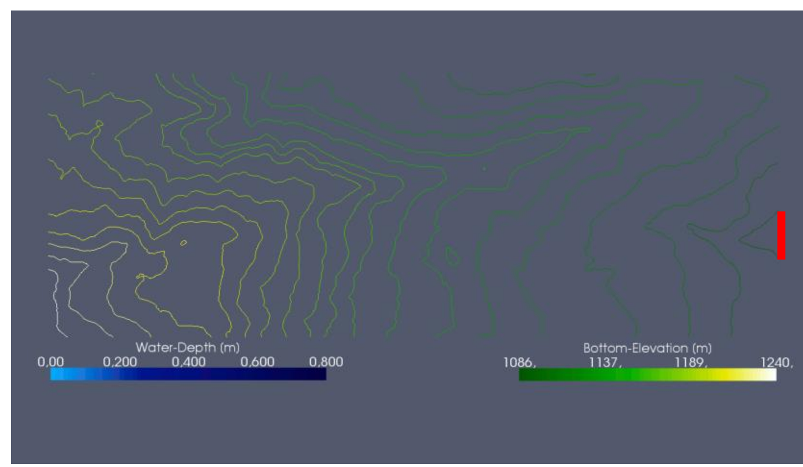


measuring weir at creek 3

comparison of simulations and measurements



high-resolution runoff simulation

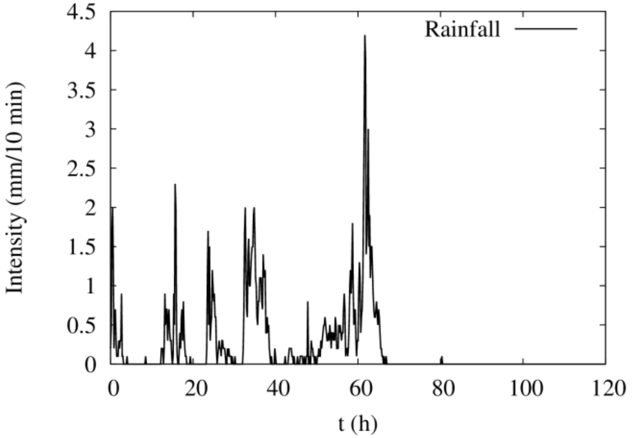


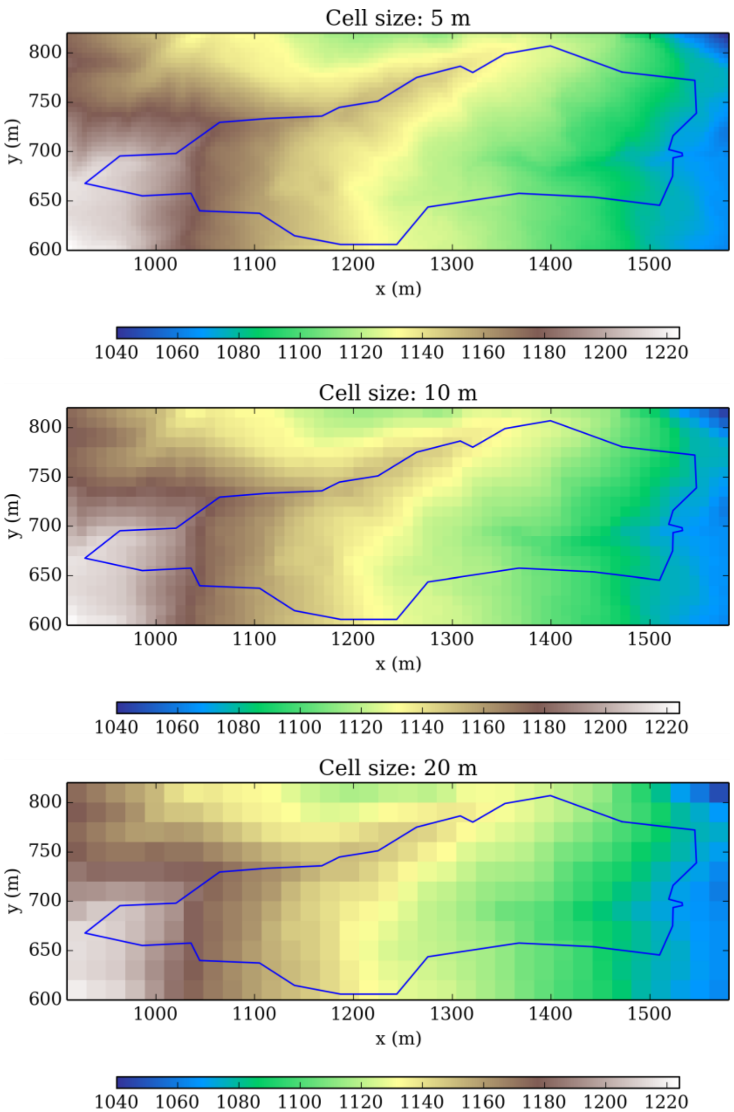
Simons *et al.* (2014)

Rainfall-runoff in a small alpine catchment (2)

- The standard deviation of the microtopography is σ = 0.20m.
- The HR model uses cells with length Δx = 1m (DEM resolution).
- All other models use cells with length Δx = 5m, which is increased to Δx = 10m in a second and to Δx = 20m in a third simulation.
- Rain intensity is applied according to a measured time series.

Rainfall-runoff in a small alpine catchment (2)



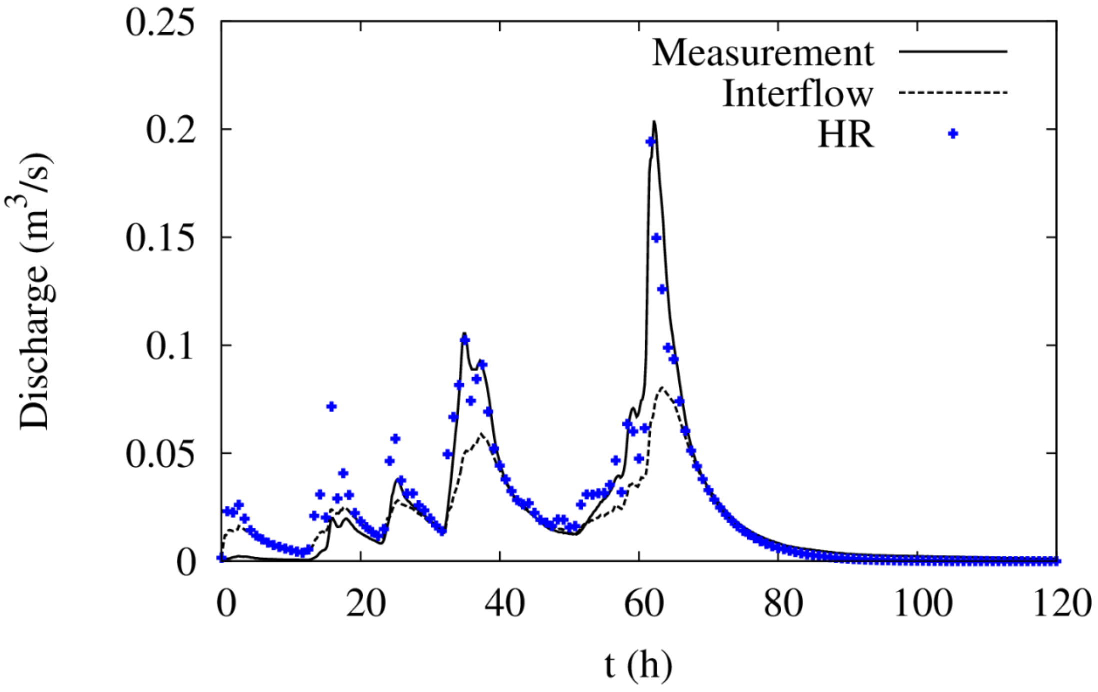


grid used for calibration

Özgen *et al.* (2015b)

Rainfall-runoff in a small alpine catchment (2)

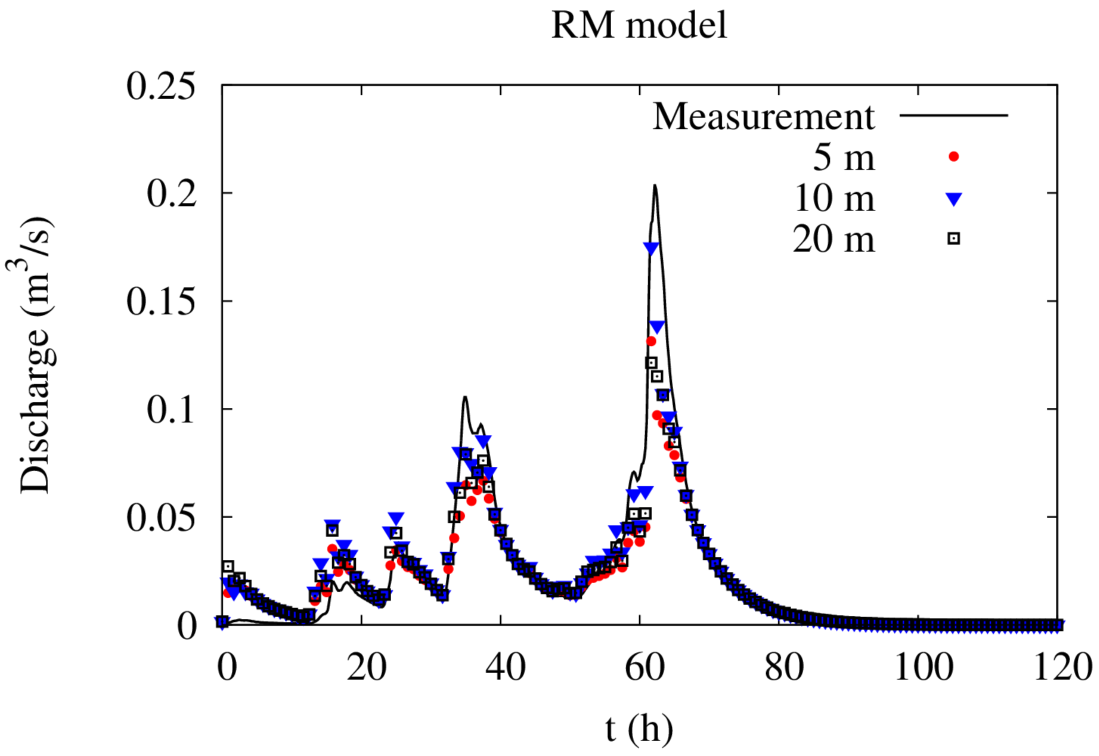
High-resolution model



Özgen *et al.* (2015b)

Rainfall-runoff in a small alpine catchment (2)

Proposed roughness approach

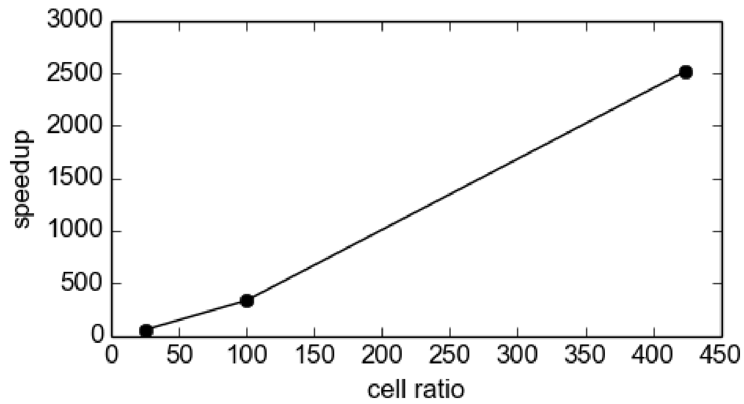


Özgen *et al.* (2015b)

Rainfall-runoff in a small alpine catchment (2)

- The table below shows the speedups
  (wall time of coarse model / wall time of high-resolution model).

Calc.	Size (HR)	Size (CR)	Nr. (HR)	Nr. (CR)	Ratio	Speedup
2a	1m	5m	147 400	5896	25	56
2b	1m	10m	147 400	1474	100	336
2c	1m	20m	147 400	374	424	2520



- **Speedup increases with cell ratio!**

Friction-based upscaling methods: Concluding remarks and outlook

- The **RM approach** shows **good agreement** of **discharge at outlet** with the high-resolution simulation results and measurements.
- A **speedup** from about **two to three orders of magnitude** was achieved.
- An alternative friction law approach with 2 calibration parameters was developed by Teuber (2015), but investigations by Özgen *et al.* (2015c) have found that the **3 parameter approach** is a bit more advantageous.
- Calculating the inundation ratio individually in each cell is a bit more advantageous.

- Numerical experiments to further validate the approach and perhaps find **upper and lower bounds for the calibration parameters** are currently carried out at the Chair of Water Resources Management and Modeling of Hydrosystems.

Anisotropic porosity-based upscaling method

Anisotropic porosity-based upscaling method

Anisotropic porosity-based upscaling method

- The anisotropic porosity method accounts for the **unresolved structures** via two types of porosity:
    1. **Volumetric porosity**: Accounts for obstructions of flow inside the cell
    2. **Areal porosity**: Accounts for obstructions of flow at the cell edges

- As this method uses two types of porosities, it differs from single porosity methods, e.g. Defina (2000), Guinot & Soares-Frazão (2006), being referred to as *isotropic porosity methods.* This method is called **anisotropic porosity method**, cf. Sanders *et al.* (2008).

- Özgen *et al.* (2016a) extended the equations of Sanders *et al.* (2008) to account for **full inundation of the computational cells**.

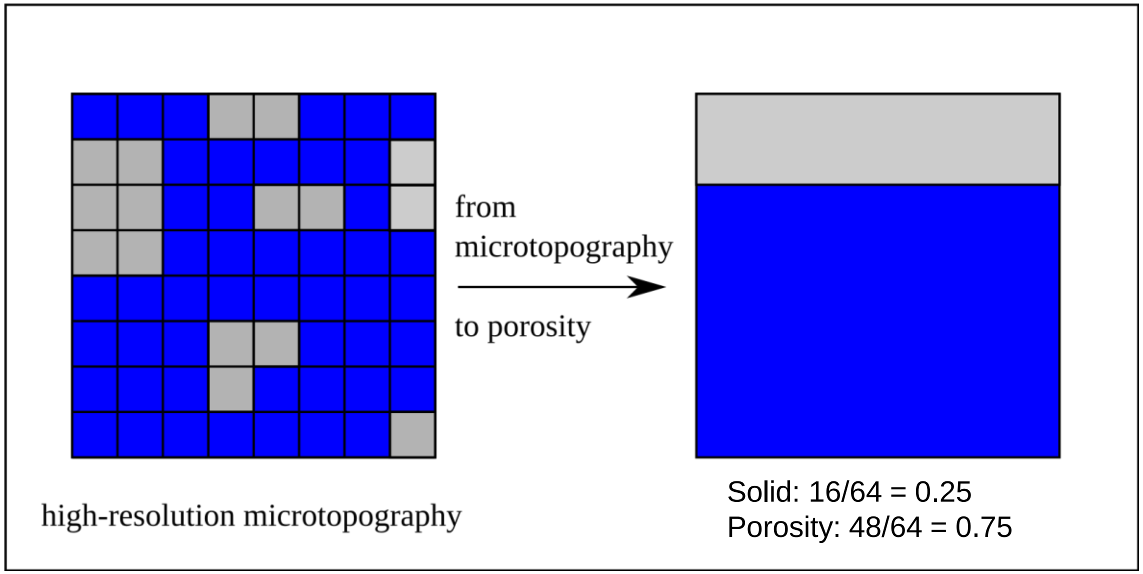
Anisotropic porosity-based upscaling method

- In the anisotropic porosity method, each cell is assigned **individual porosity** functions which are based on the distribution function of the unresolved structures.

  – Physically based, intuitive method (+)
  – Good level of accuracy without extensive calibration (+)

  – Due to its formulation, only solvable with the Finite-Volume Method (-)
  – Modification of every term in the shallow water equations necessary (-)

Anisotropic porosity-based upscaling method

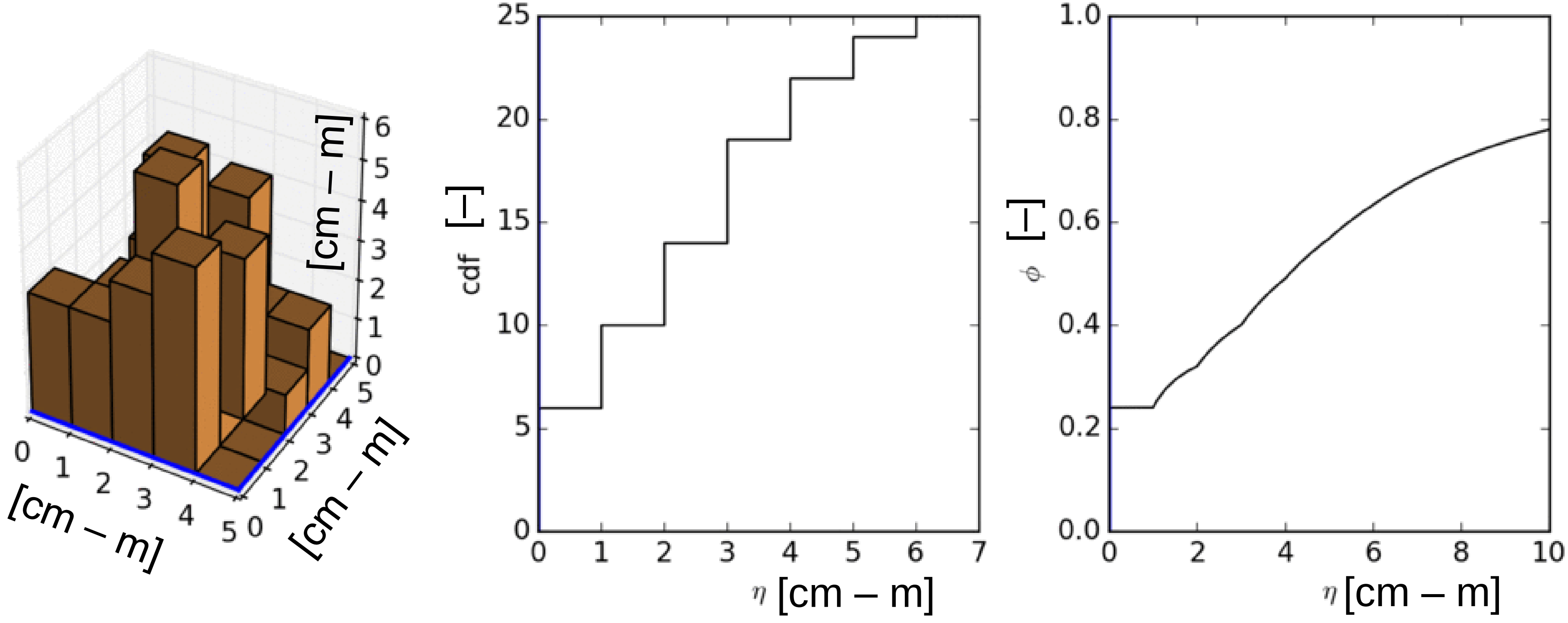


blue colour: water,
grey colour: solid

- Microtopography and buildings influence the flow in the cell.
- The cell can be considered as a porous medium and the microtopography or buildings can be taken into account as a porosity.

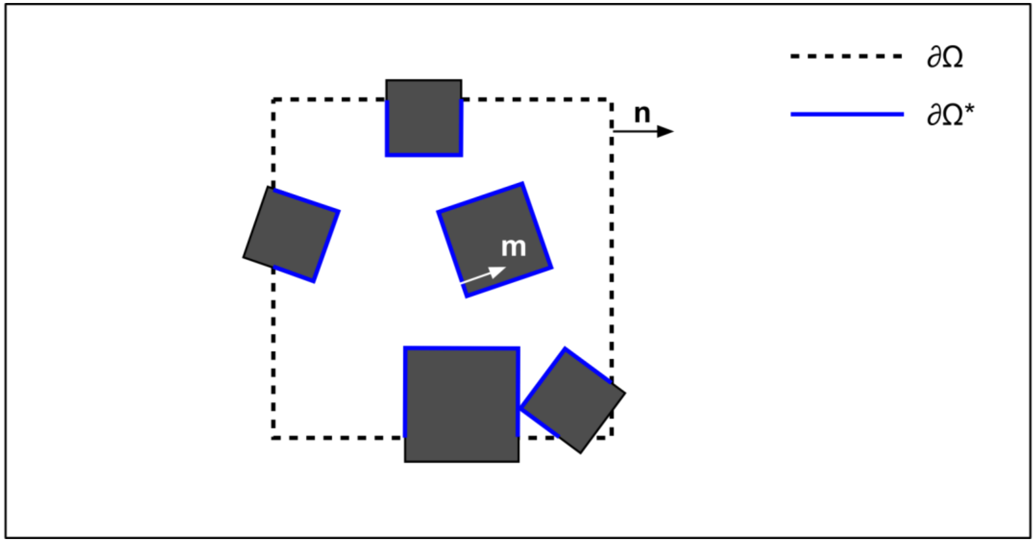
Anisotropic porosity-based upscaling method

- Volumetric porosity calculation:



Anisotropic porosity-based upscaling method

- In this figure the blue line illustrates the interface between the fluid and the unresolved blocks inside the cell, whereby the dashed line is the boundary of the cell.



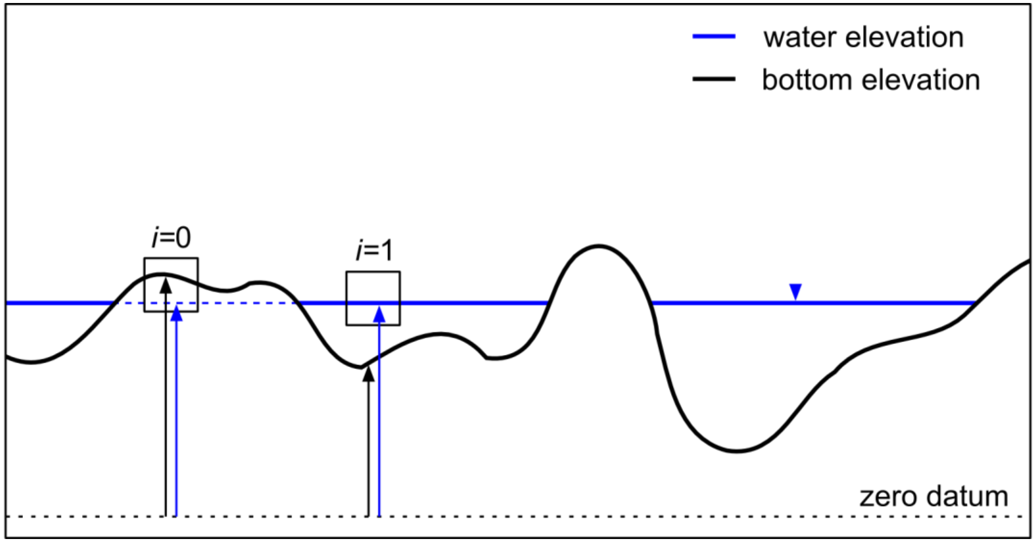
Özgen *et al.* (2016a)

Anisotropic porosity-based upscaling method

- For the derivation of the equations, the **phase function** *i* is introduced:

$$i = \begin{cases} 0, & \text{if } z_b > \eta \\ 1, & \text{else} \end{cases}$$

where z_b is the bottom elevation and η is the free surface water elevation.



Özgen *et al.* (2016a)

Anisotropic porosity-based upscaling method

- Then, porosities are defined as:

$$\phi = \frac{\int_{\Omega} i(\eta - z_b)\, d\Omega}{\int_{\Omega} (\eta - z_0)\, d\Omega}$$

$$\Psi = \frac{\oint_{\partial\Omega} i(\eta - z_b)\, dr}{\oint_{\partial\Omega} (\eta - z_0)\, dr}$$

whereby z_0 is the zero datum.

- In words, they stand for the "**ratio of the volume available for the flow to the total volume of the cell**" and the "**ratio of the area available for the flow to the total area of the cell edge**", respectively.

- Substituting the porosities into the integral shallow water equations gives the flux and source vectors of the shallow water equations with anisotropic porosity and the modified Finite-Volume expression, cf. Özgen *et al.* (2016a).

Governing equations

- **Multiplying every term** (except the source term) of the shallow water equations gives:

$$i\frac{\partial \vec{q}}{\partial t} + i\frac{\partial \vec{f}}{\partial x} + i\frac{\partial \vec{g}}{\partial y} = \vec{s}$$

$$\vec{q} = \begin{bmatrix} h \\ q_x \\ q_y \end{bmatrix}, \quad \vec{f} = \begin{bmatrix} q_x \\ v_x q_x + 0.5gh^2 \\ v_x q_y \end{bmatrix}, \quad \vec{g} = \begin{bmatrix} q_y \\ v_y q_x \\ v_y q_y + 0.5gh^2 \end{bmatrix}, \quad \vec{s} = \begin{bmatrix} -s_m \\ s_{b,x} + s_{f,x} \\ s_{b,y} + s_{f,y} \end{bmatrix}$$

Governing equations

- In the following, the **integal conservative formulation** of the shallow water equations is used to derive the porosities.

- The integral conservative formulation can be obtained by applying mass and momentum conservation laws to a fixed Eulerian control volume under the assumption of hydrostatic pressure:

$$\frac{\partial}{\partial t}\int_{\Omega} i\vec{q}\,d\Omega + \oint_{\partial\Omega} i\vec{F}\vec{n}\,dr = \int_{\Omega} \vec{s}\,d\Omega + \boxed{\oint_{\partial\Omega^*} \vec{s}^{\,*}\,dr}$$

Ω is the control volume and $\partial\Omega$ is the control volume boundary.

- $\vec{s}*$ results from the macroscopic model concept and is a source vector accounting for fluid **pressure along the interface between fluid and** (subgrid-scale) **solid**.

Governing equations

- The Finite-Volume formulation becomes:

$$\phi^{n+1}\vec{q}^{\,n+1} = \phi^{n}\vec{q}^{\,n} - \frac{\Delta t}{A}\sum_k \psi^{n}\vec{F}_k^{\,n}\vec{n}_k l_k + \Delta t\left(\vec{s}^{\,n} + \oint_{\partial\Omega^*}\vec{s}^{\,*}dr\right)$$

$$\vec{q} = \begin{bmatrix} \overline{\eta} - z_0 \\ \overline{u}\left(\overline{\eta} - z_0\right) \\ \overline{v}\left(\overline{\eta} - z_0\right) \end{bmatrix}, \quad \vec{F}\vec{n} = \begin{bmatrix} \widehat{u}\left(\hat{\eta} - z_0\right)n_x + \hat{v}\left(\hat{\eta} - z_0\right)n_y \\ \widehat{u}\widehat{u}\left(\hat{\eta} - z_0\right)n_x + 0.5g\hat{h}\left(\hat{\eta} - z_0\right)n_x + \widehat{u}\hat{v}\left(\hat{\eta} - z_0\right)n_y \\ \hat{v}\widehat{u}\left(\hat{\eta} - z_0\right)n_x + 0.5g\hat{h}\left(\hat{\eta} - z_0\right)n_x + \hat{v}\hat{v}\left(\hat{\eta} - z_0\right)n_y \end{bmatrix}$$

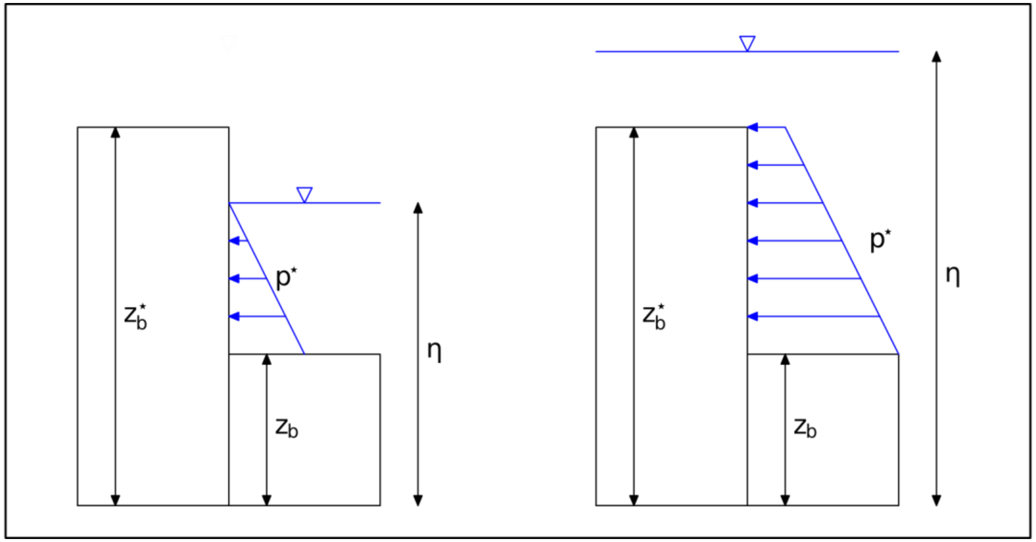
where the bar denotes a “volume-averaged“ variable and the circumflex denotes an “area-averaged“ variable.

- **Porosities are updated every time step** depending on the water level changes.

Governing equations

- The **interfacial pressure source term** *s** is calculated as the sum of a stationary component and a non-stationary component.
- The **stationary component** can be calculated as:

$$\vec{s}_{st}^{\,*} = \begin{bmatrix} 0 \\ p^* m_x \\ p^* m_y \end{bmatrix}, \quad p^* = \begin{cases} 0.5g\left(\eta - z_b\right)^2 & \text{if } \eta \leq z_b^* \\ 0.5g\left(\left(\eta - z_b^*\right) + \left(\eta - z_b\right)\right)\left(z_b^* - z_b\right) & \text{else} \end{cases}$$



Özgen *et al.* (2016a)

Governing equations

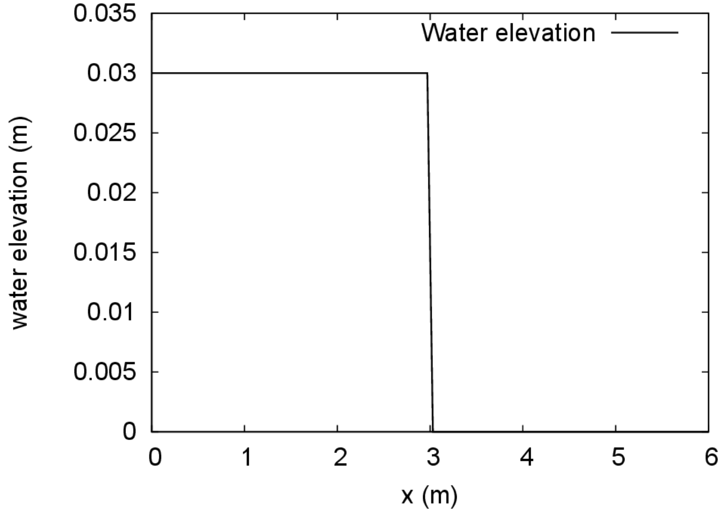
- The **non-stationary pressure source term** is usually approximated by a drag force law as:

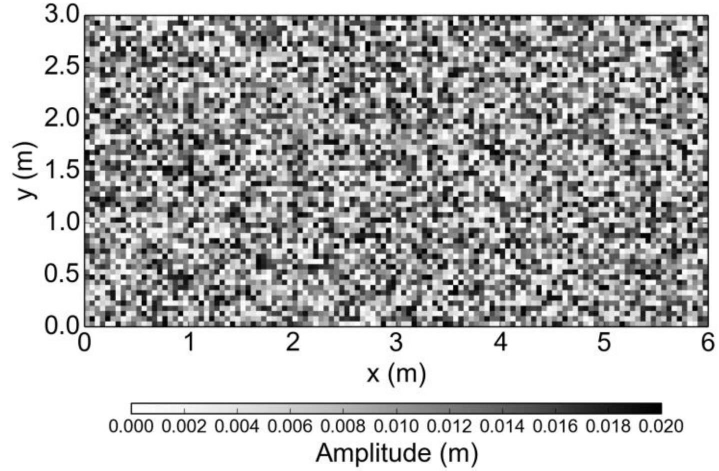
$$\vec{s}_{nst}^{\;*} = \begin{bmatrix} 0 \\ c_D u|\vec{v}| \\ c_D v|\vec{v}| \end{bmatrix}, \quad c_D = 0.5 c_D^0 a \min\left(h, z_b^* - z_b\right)$$

Here, *a* [m²] stands for the area the drag force is acting on, c_D [-] and c_D^0 [m^{-3}] stand for **drag coefficients**. *a* is very difficult to determine, as it not only depends on the topography but also on the flow direction. Therefore, the model in this study is **calibrated with the product** $\boldsymbol{a \cdot c_D^0}$ [m^{-1}].

Dam break on bed with random microtopography (4)

- Similar to the first dam break, the initial conditions are $\eta_0 = 0.03$m on the left side and dry bed on the right side of the discontinuity. The microtopography of this case is plotted in the figure below (bed varies between 0 and 2cm):



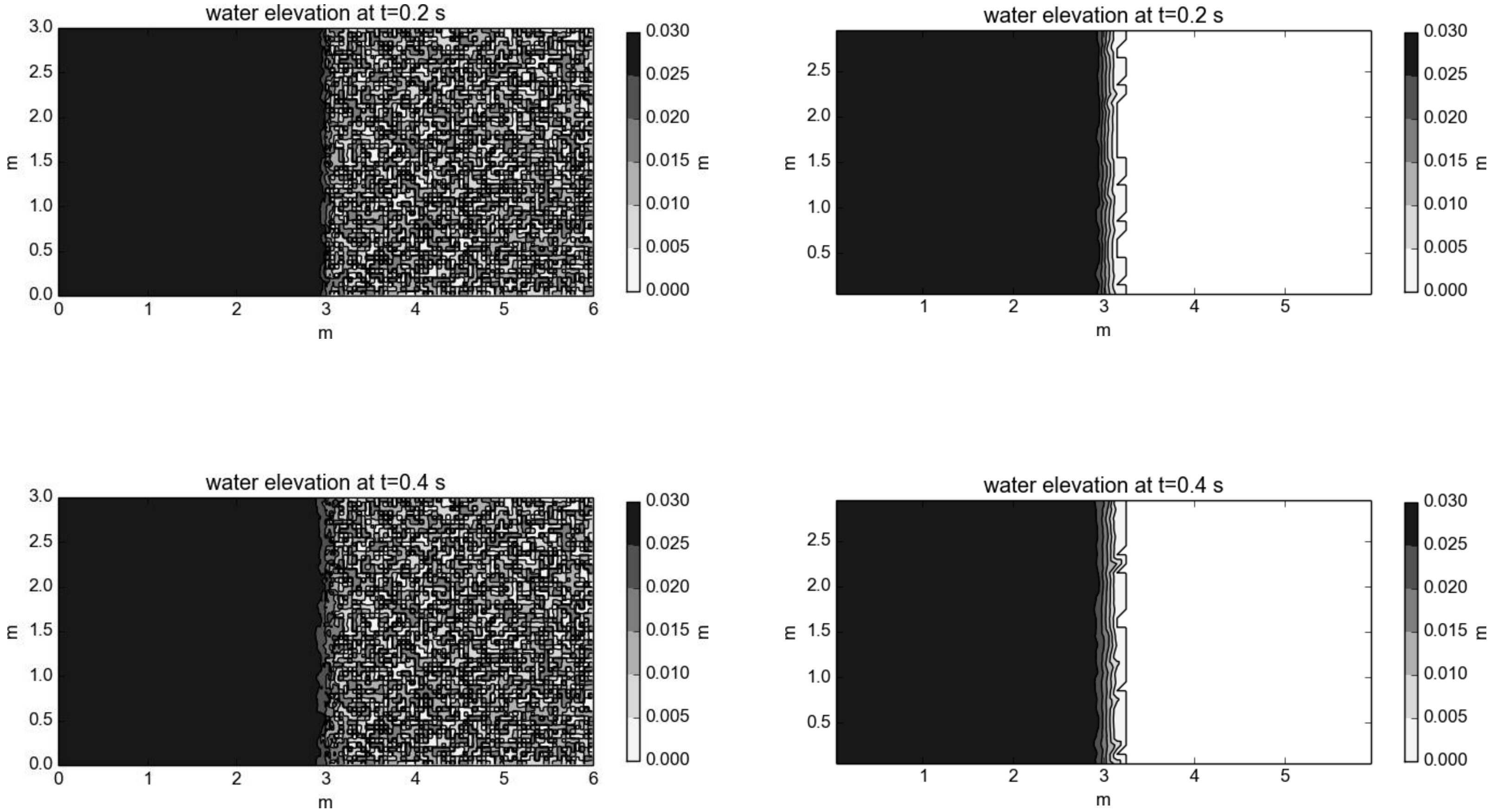


- A high-resolution model with $\Delta x = 0.01$m is used as reference, the anisotropic porosity model uses a grid size of $\Delta x = 0.1$m. Manning's friction coefficient is n = 0.016 sm$^{-1/3}$. In AP model drag term is $ac_D = 0$.

Özgen *et al.* (2016a)

Dam break on bed with random microtopography (4)

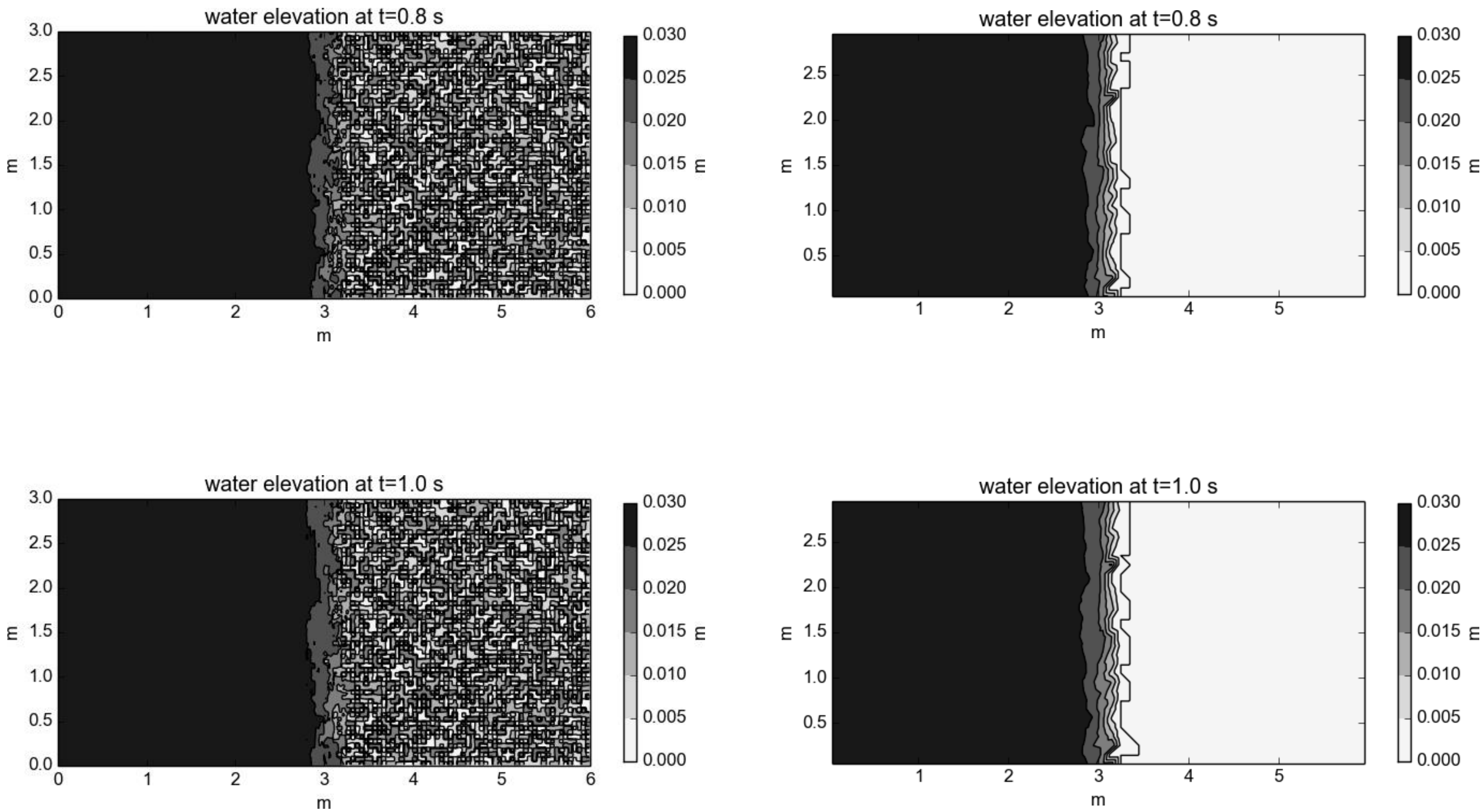
- Results at different times (high-resolution left, anisotropic porosity model right):



Özgen *et al.* (2016a)

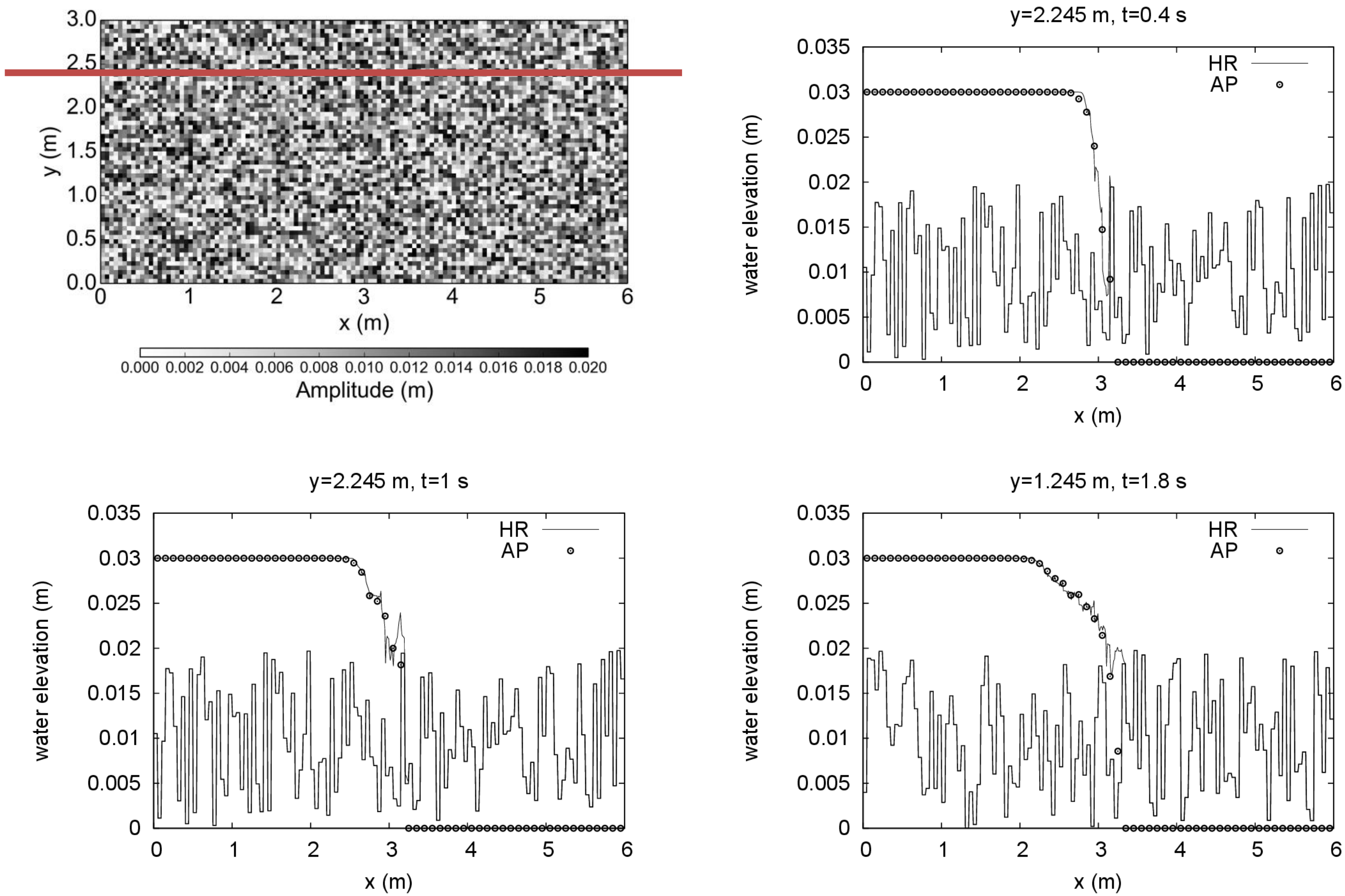
Dam break on bed with random microtopography (4)

- Results at different times (high-resolution left, anisotropic porosity model right):



Özgen *et al.* (2016a)

Dam break on bed with random microtopography (4)



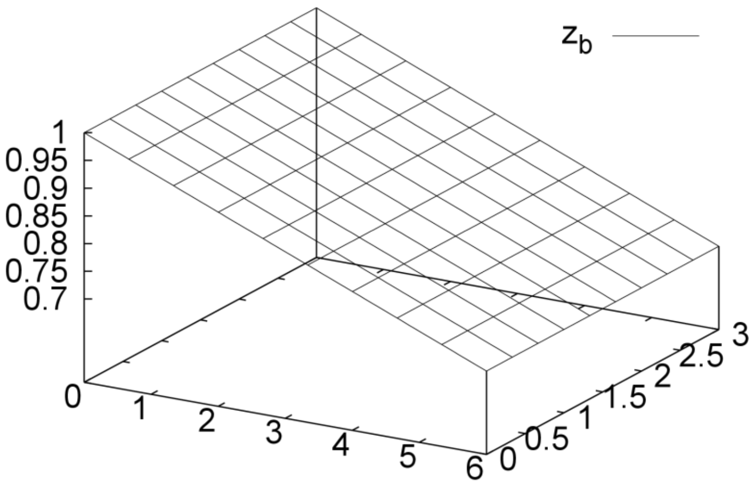
Özgen *et al.* (2016a)

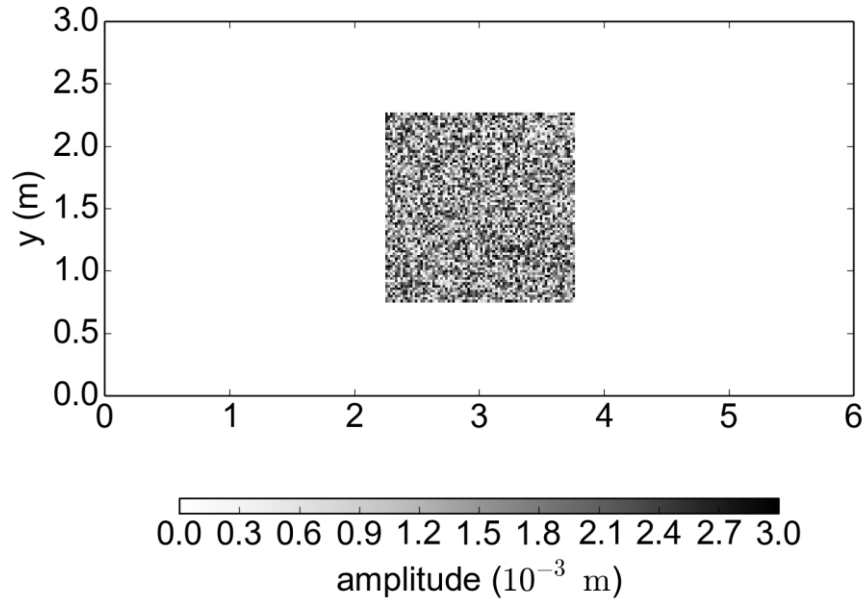
Dam break on bed with random microtopography (4)

- An overall **speedup of about 1000** has been achieved.

Rainfall-runoff on an inclined plane with random microtopography (5)

- The geometry of the inclined plane is as illustrated below:





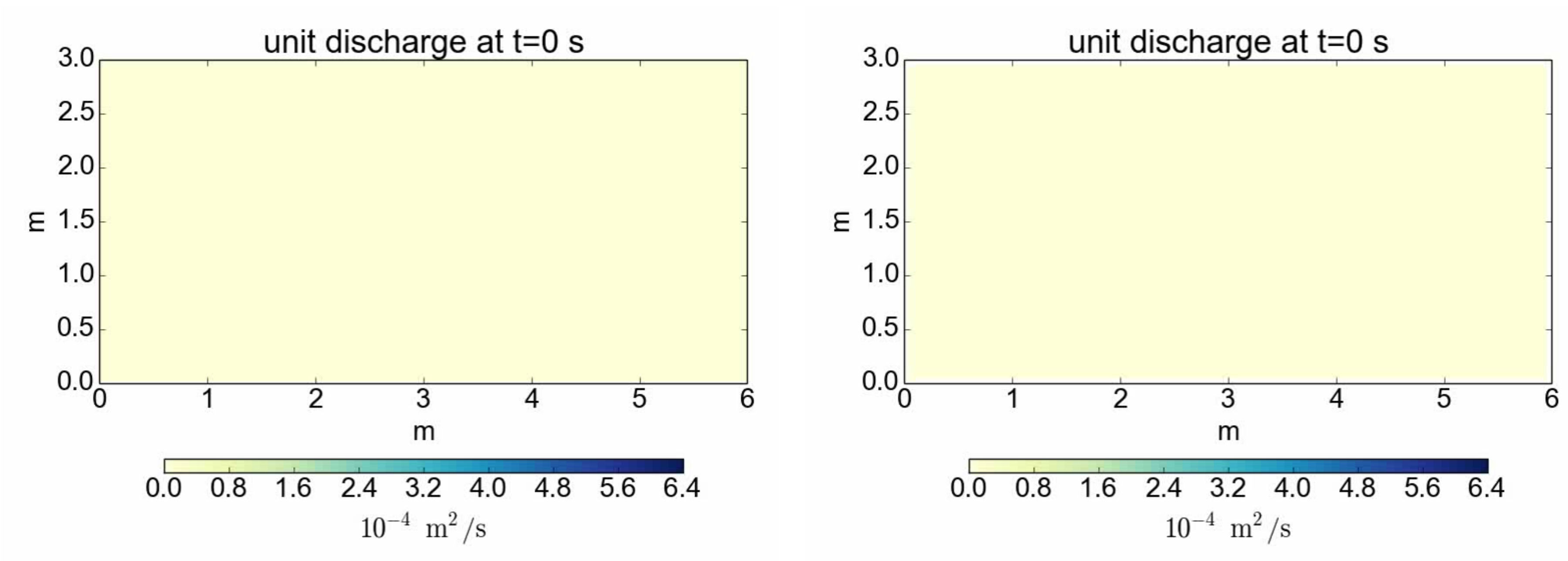
Özgen *et al.* (2016a)

Rainfall-runoff on an inclined plane with random microtopography (5)

- A high-resolution model with Δx = 0.02m is used as reference, the anisotropic porosity model uses a grid size of Δx = 0.1m. Manning's friction coefficient is n = 0.016 $sm^{-1/3}$. In AP model drag term is $ac_D = 0$.

- Constant rainfall with an intensity of i = 10^{-4}m/s is applied.

Rainfall-runoff on an inclined plane with random microtopography (5)

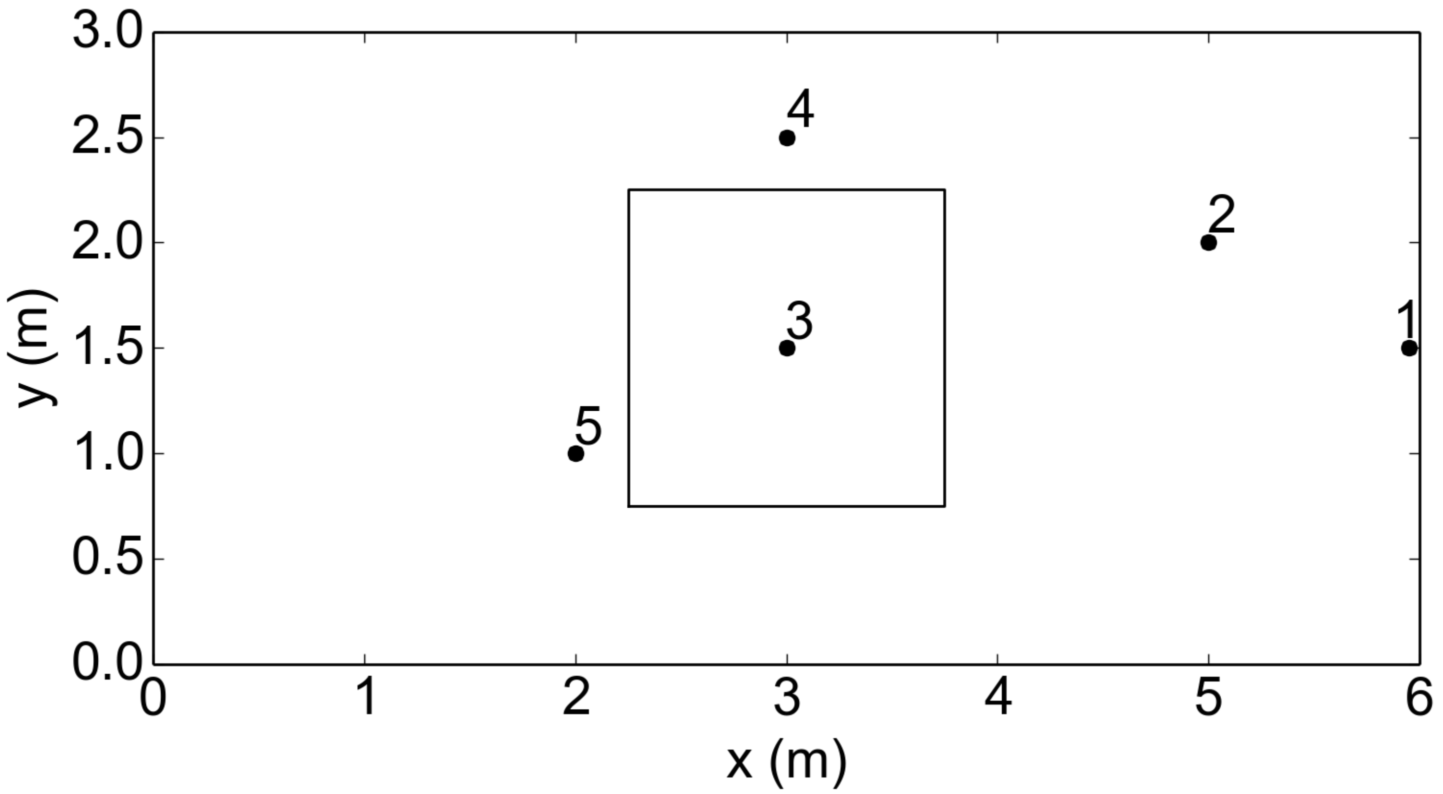


High resolution simulation (left), anisotropic porosity simulation (right)

Özgen *et al.* (2016a)

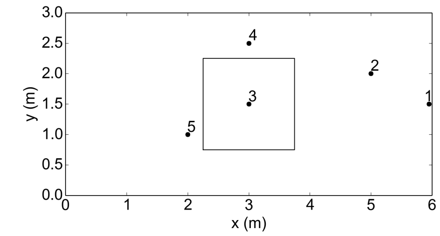
Rainfall-runoff on an inclined plane with random microtopography (5)

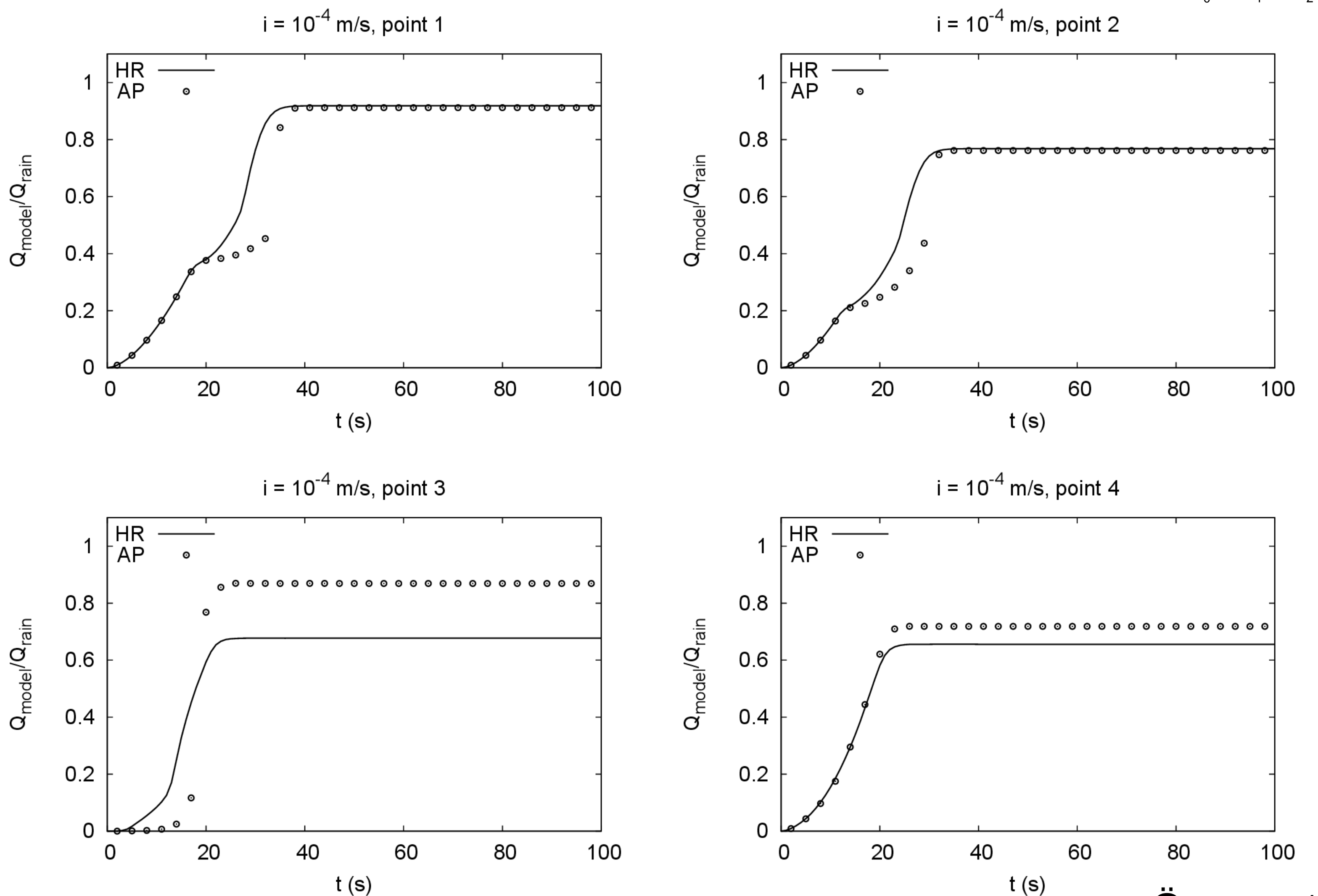
- Results are evaluated at 5 points:



Özgen *et al.* (2016a)

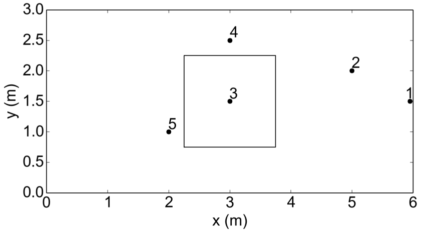
Rainfall-runoff on an inclined plane with random microtopography (5)

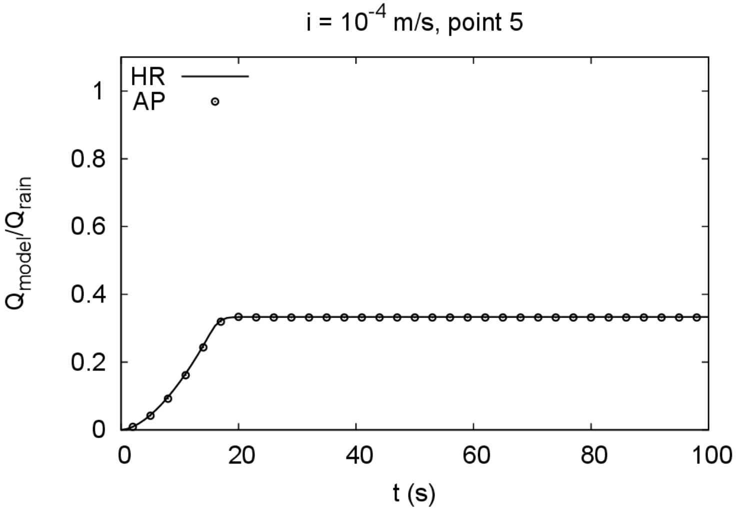


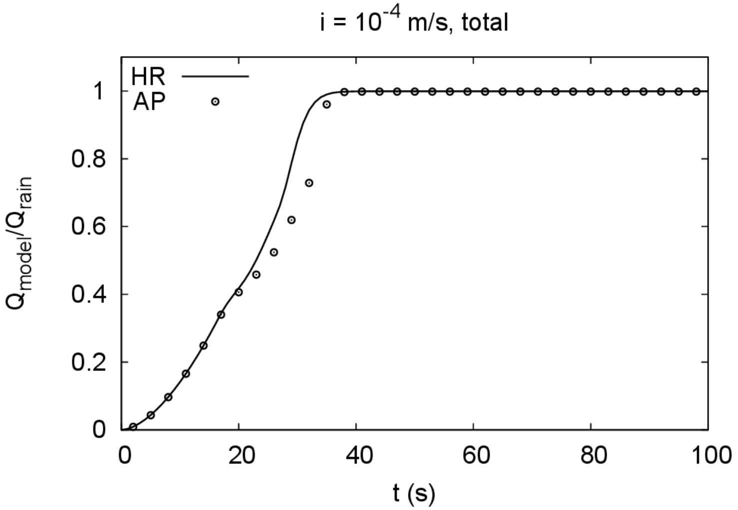


Özgen *et al.* (2016a)

Rainfall-runoff on an inclined plane with random microtopography (5)







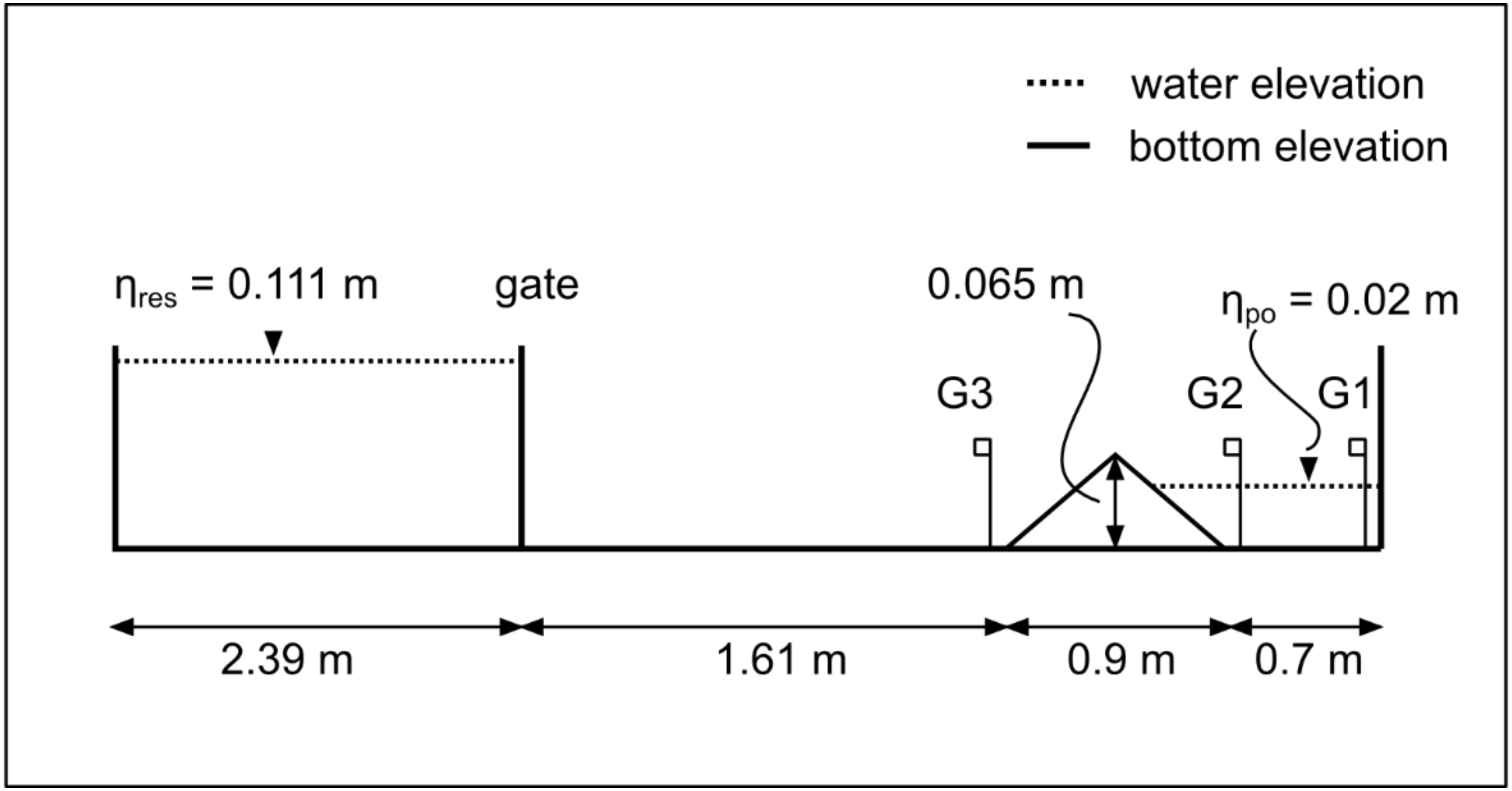
total discharge at the outlet

- A **speedup of about 550** has been obtained.

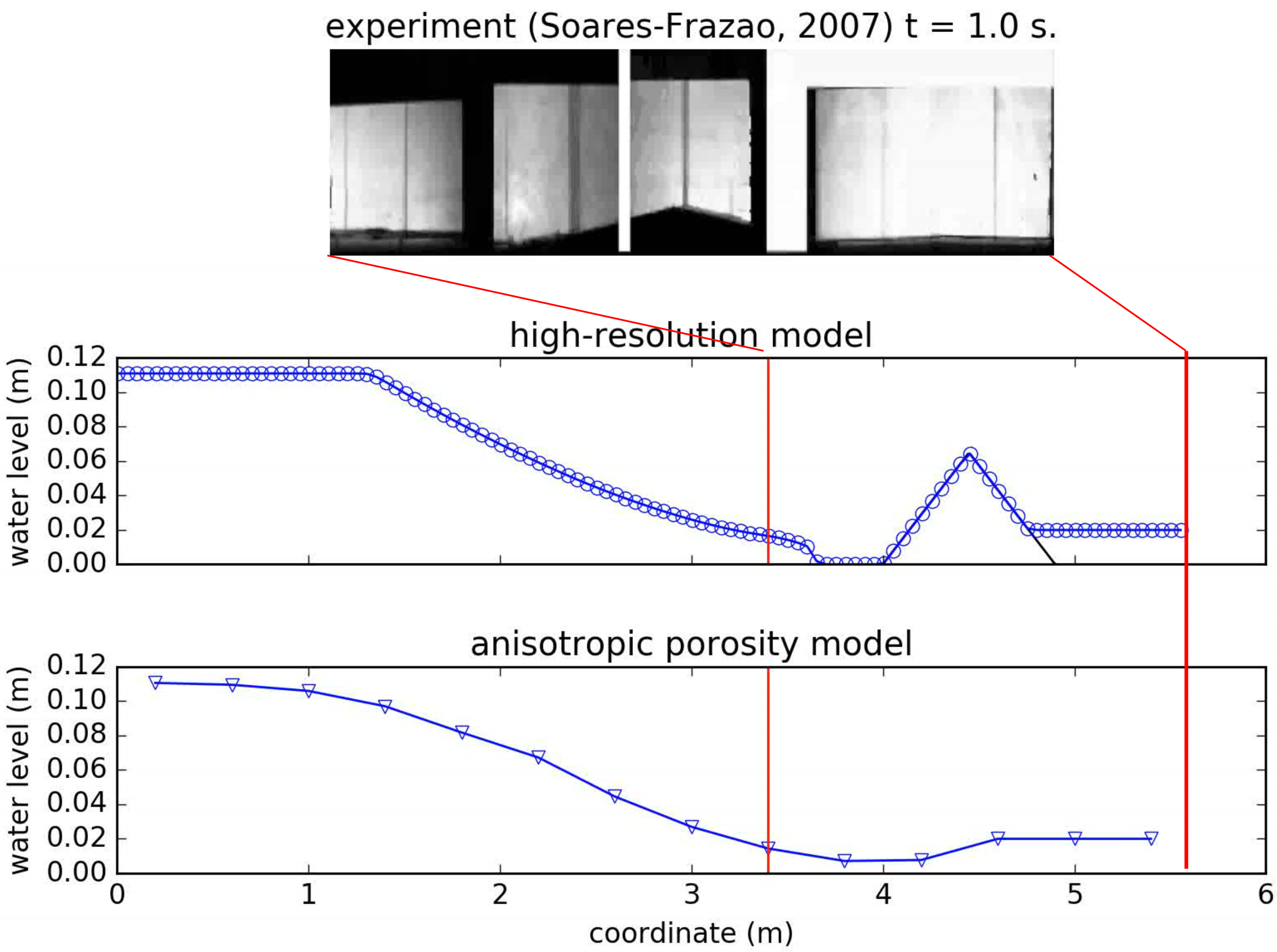
Özgen *et al.* (2016a)

Dam-break flow over sill (6)

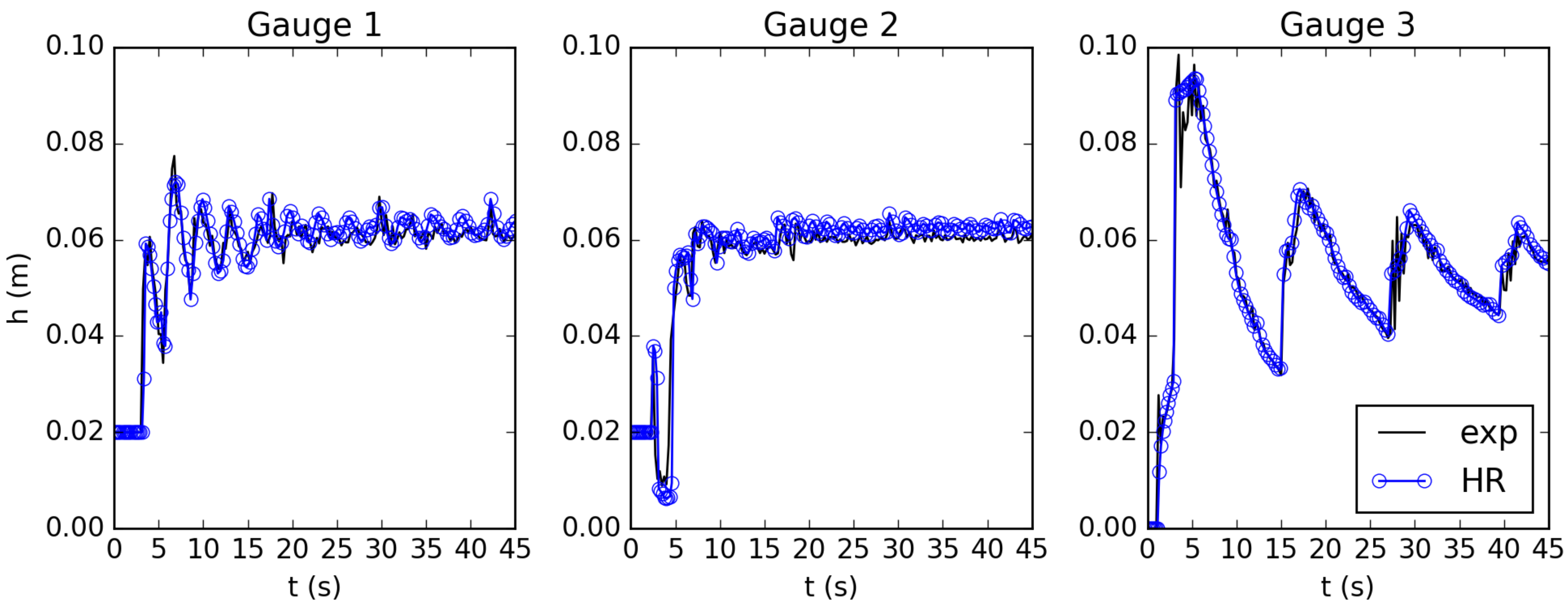
- Experiment conducted by Soares-Frazao (2008)
- HR model: $\Delta x = 0.01$m, AP model $\Delta x = 0.4$m
- Manning coefficient: $n = 0.011 \text{sm}^{-1/3}$
- AP model $ac_D = 5.0\text{m}^{-1}$



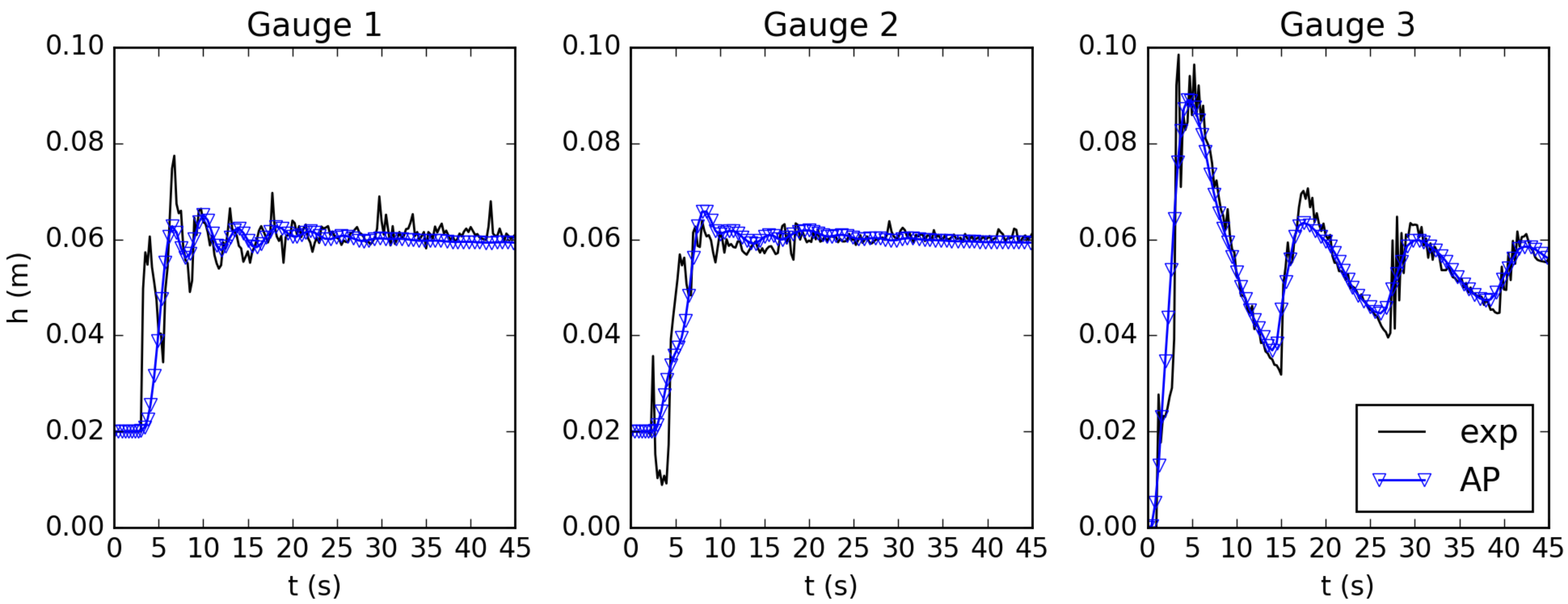
Dam break flow over sill (6)



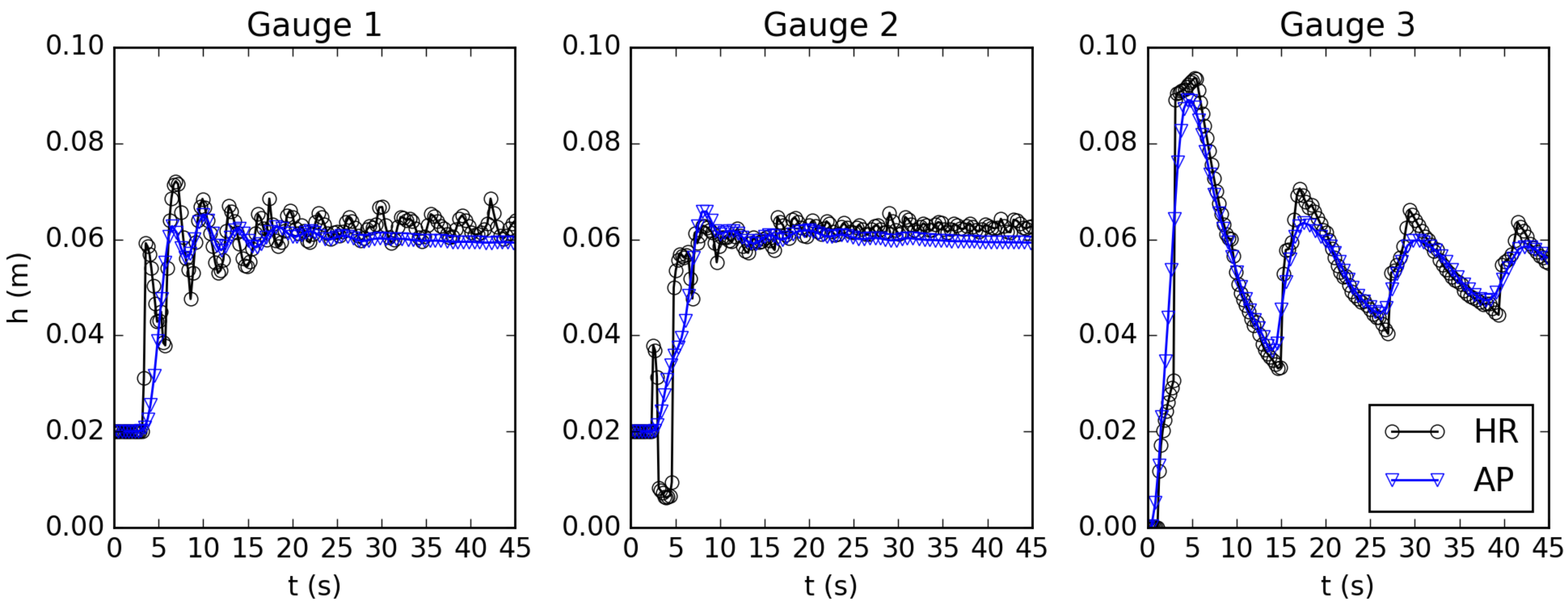
Dam-break flow over sill (HR vs experiment) (6)



Dam-break flow over sill (AP vs experiment) (6)



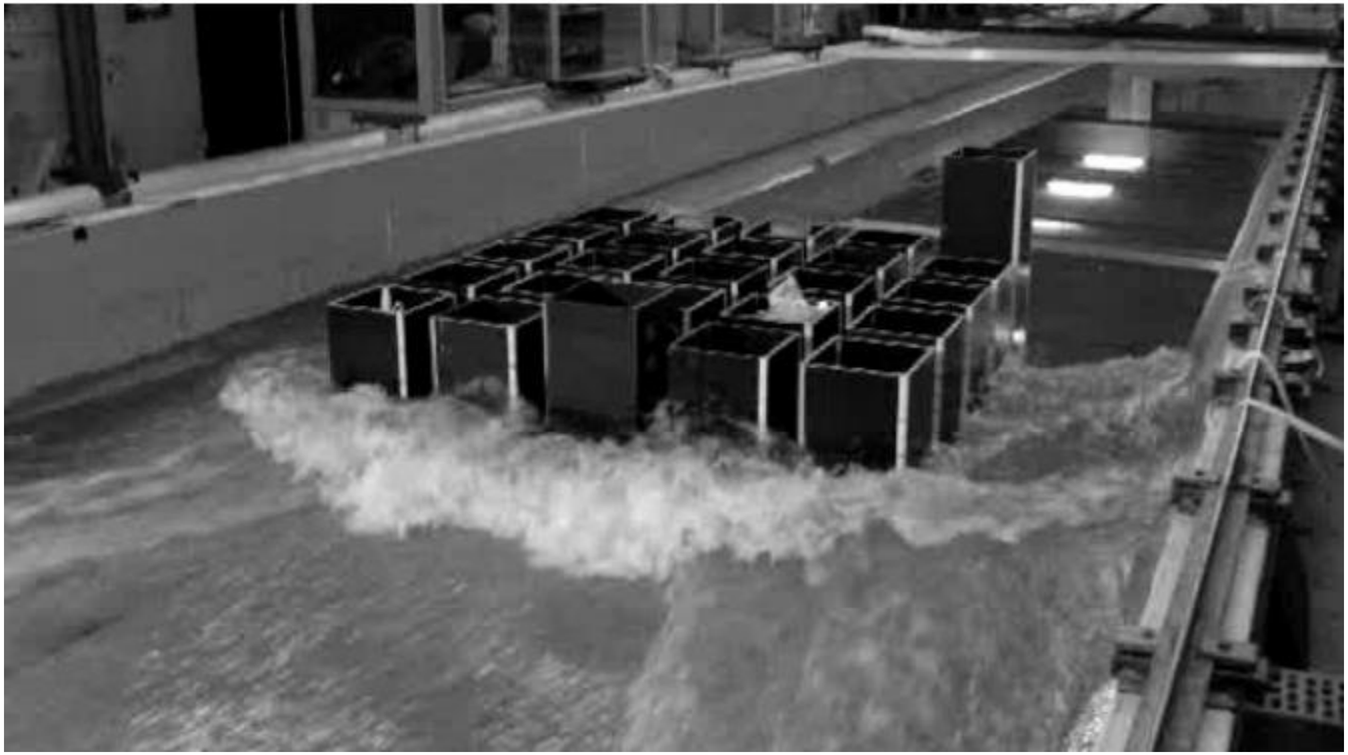
Dam-break flow over sill (AP vs HR) (6)



A speedup of about 1140 is obtained.

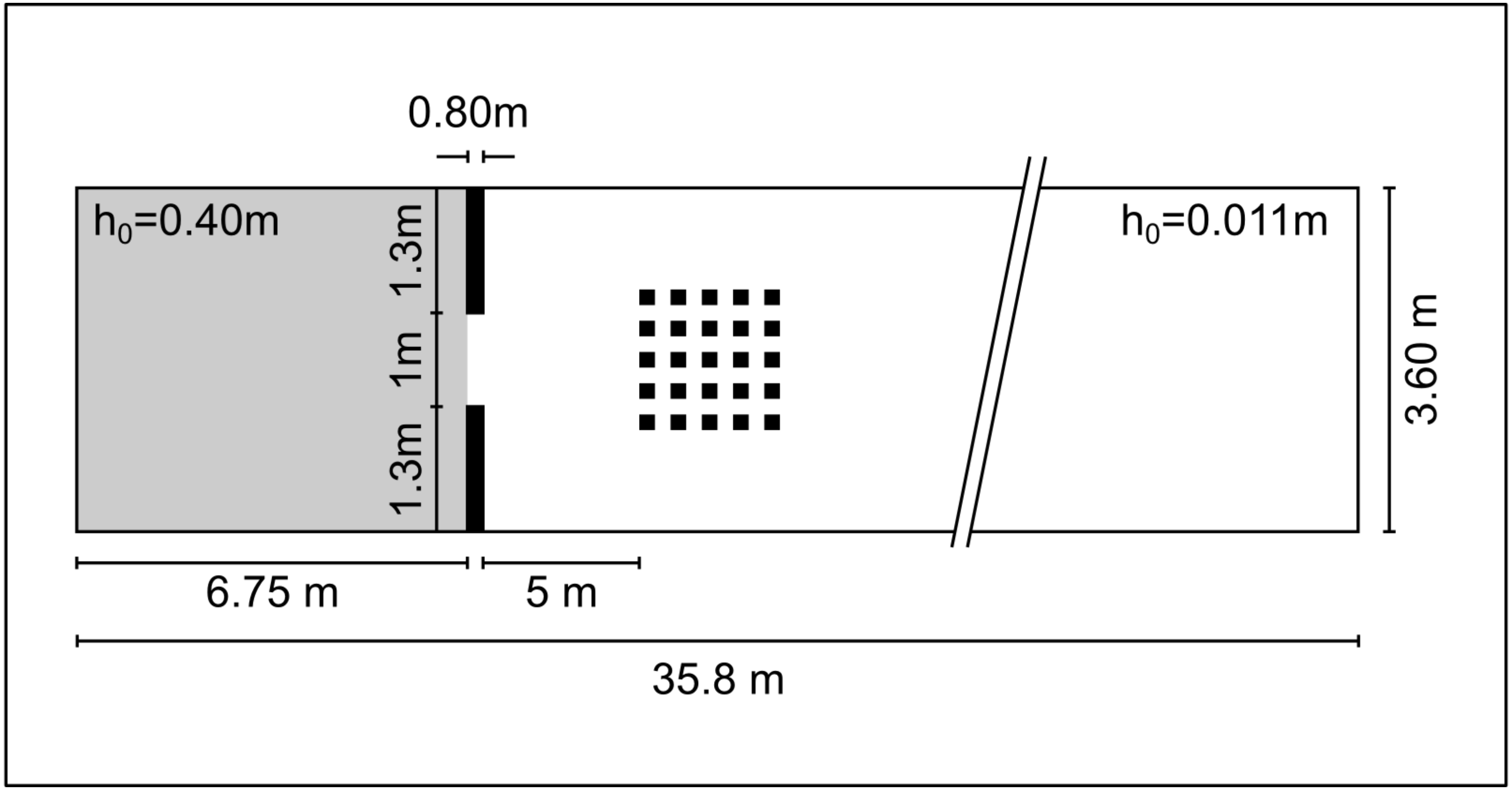
Dam-break flow through idealized city (7)

Experiment conducted by Soares-Frazao & Zech (2008)

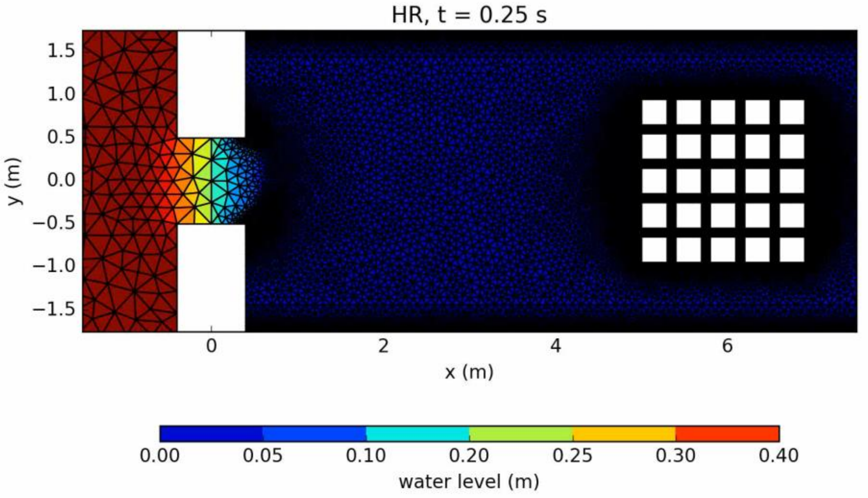


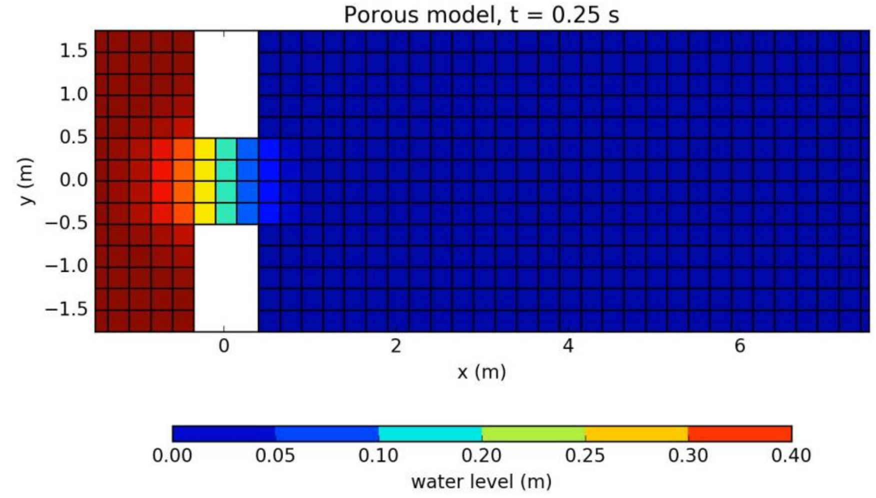
Dam-break flow through idealized city (7)

- Özgen *et al.* (2016d), HR model: Δx = 0.01-0.30m, AP model: Δx = 0.25m
- Manning coefficient: $n = 0.010 sm^{-1/3}$
- AP model $ac_D = 1.9 m^{-1}$

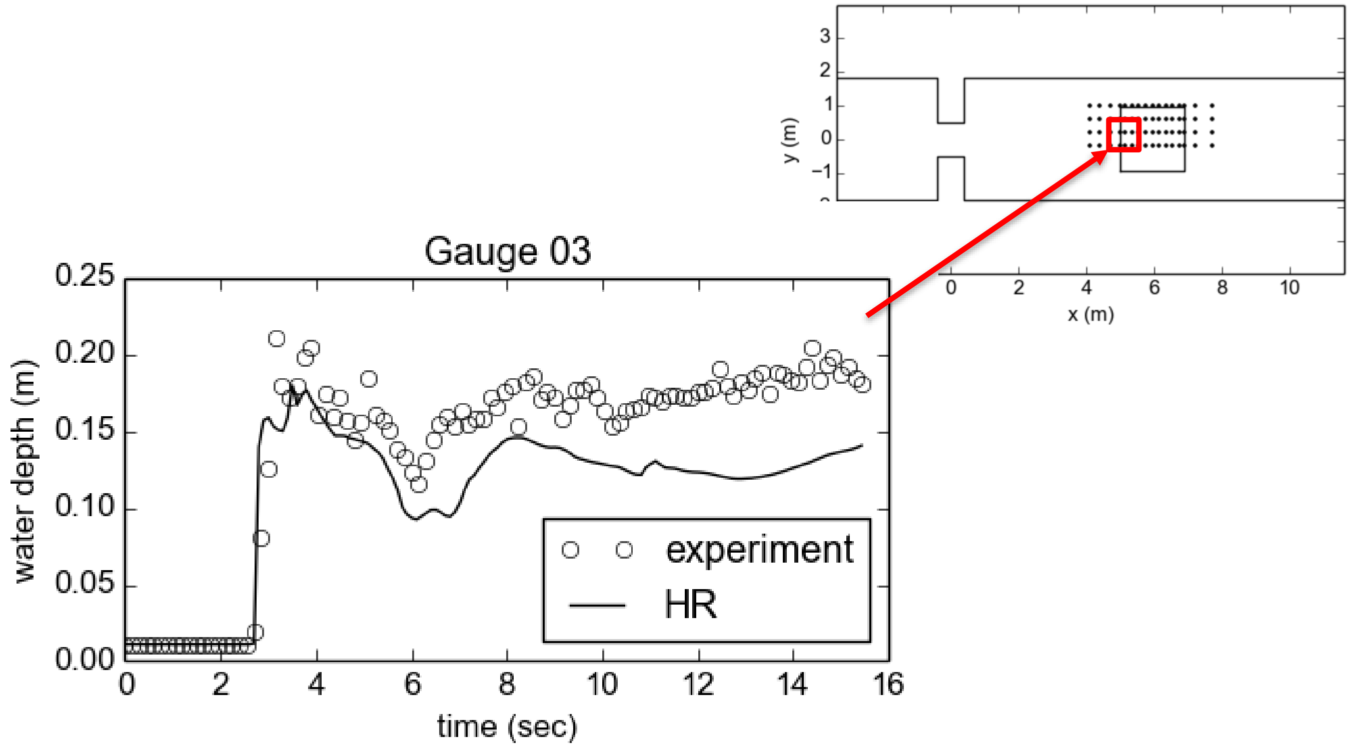


Dam break flow through idealized city (7)



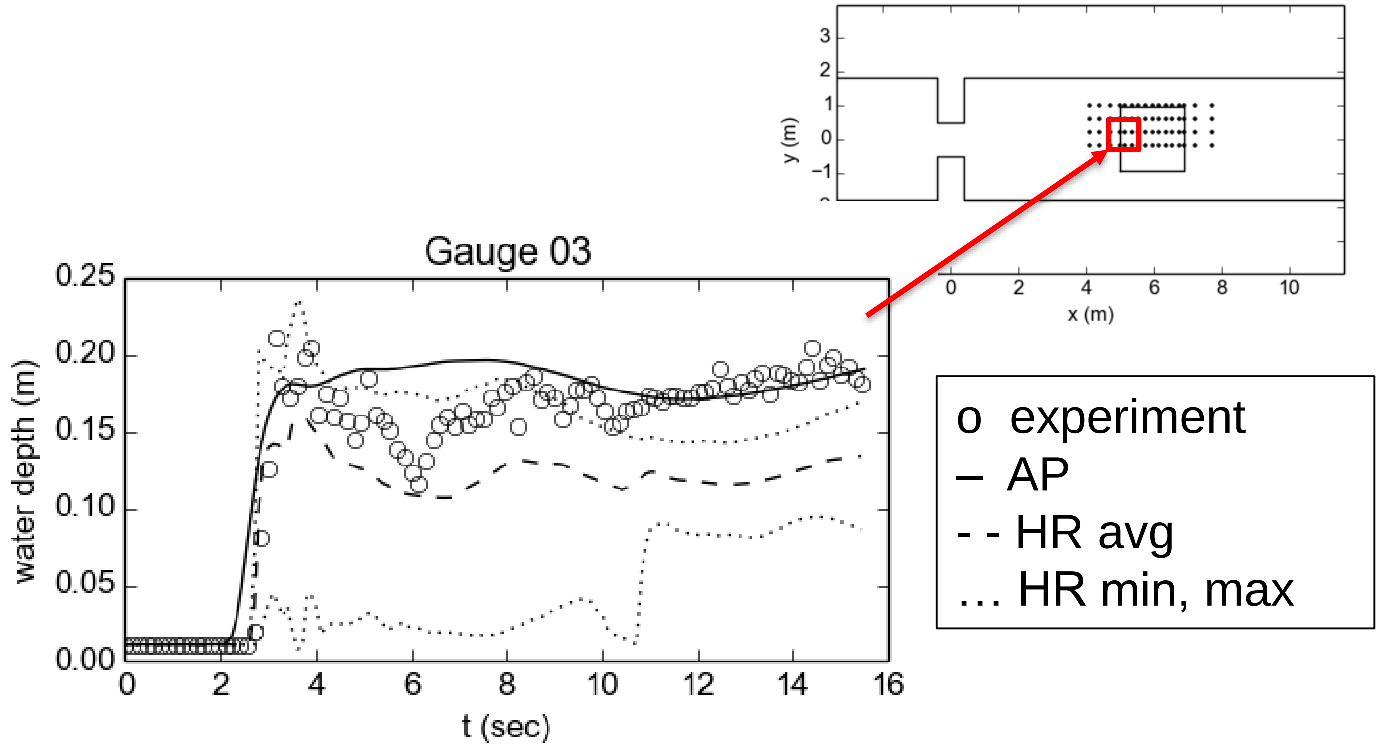


Dam-break flow through idealized city (7)



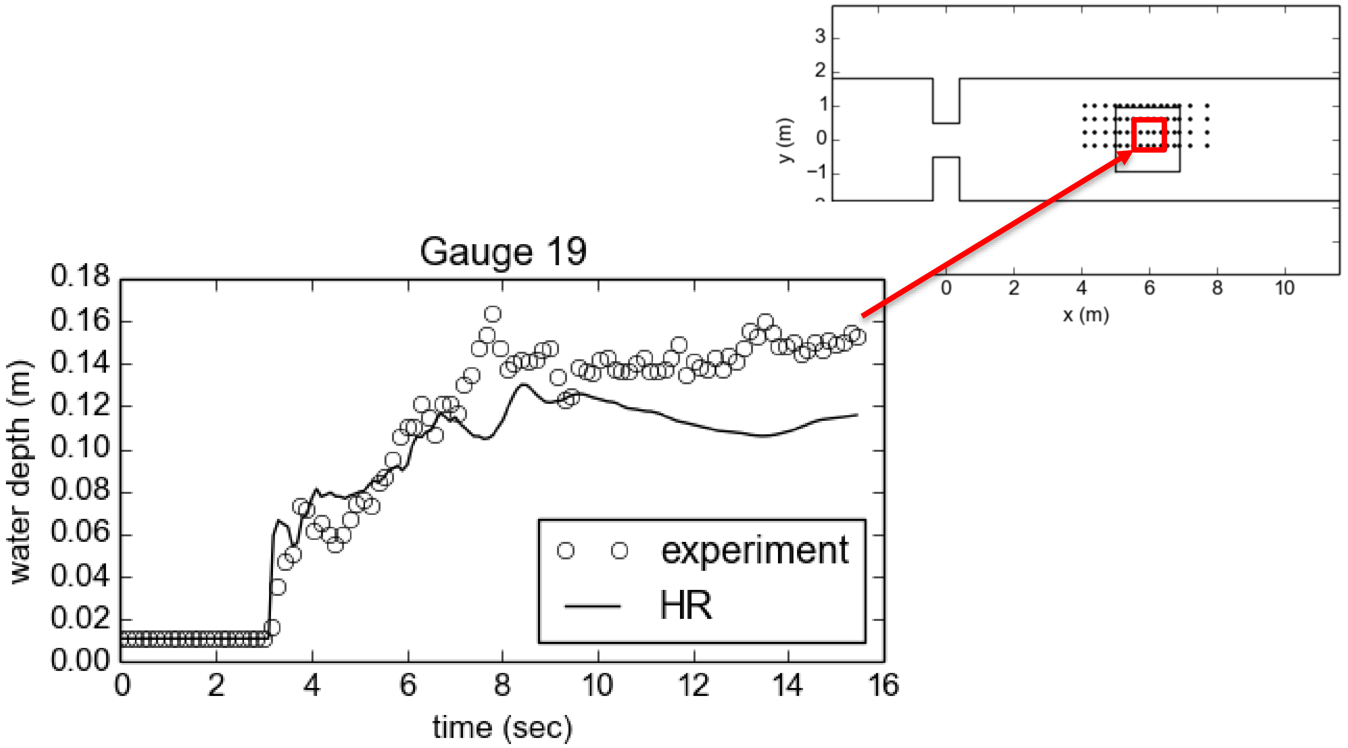
High-resolution model results (HR, Δx = 0.01m) vs experimental data

Dam-break flow through idealized city (7)



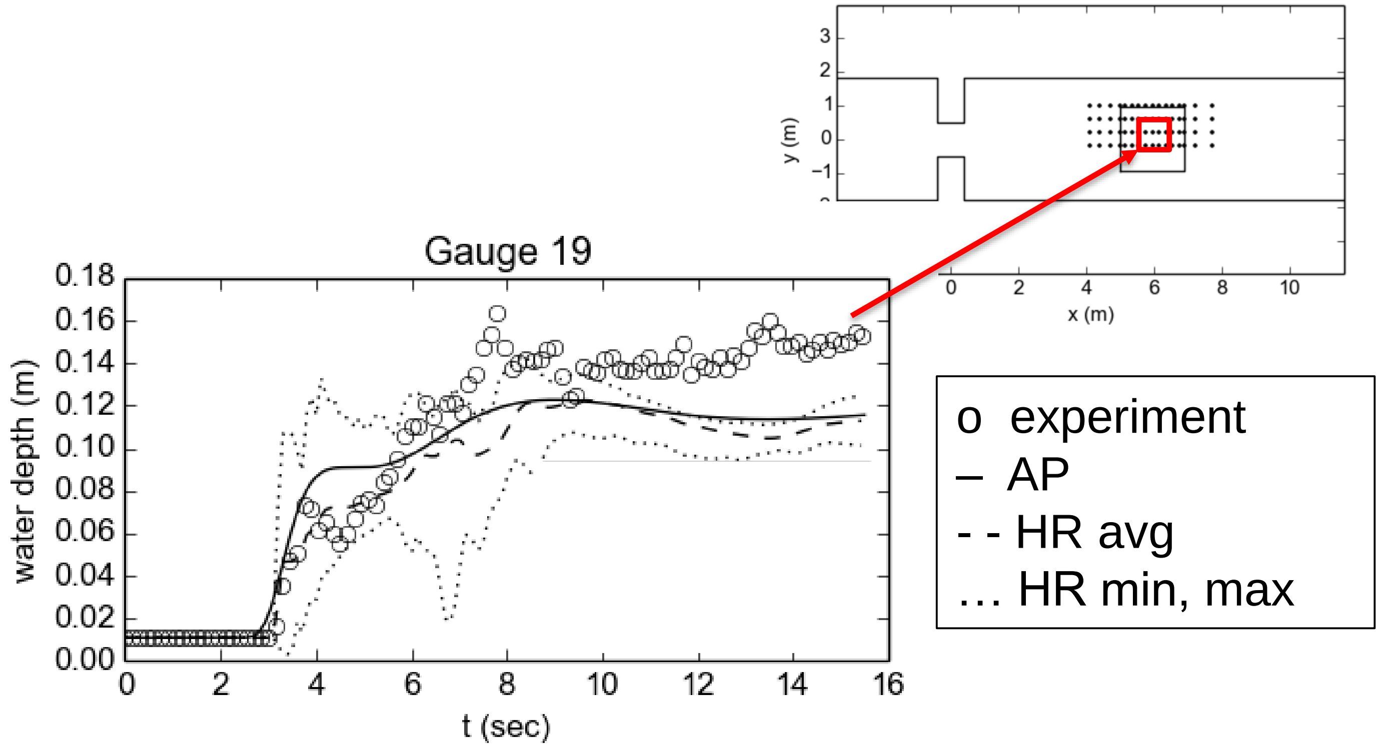
Anisotropic porosity model results (AP, Δx = 0.1m) vs experiment
vs HR model averaged in AP cells (Δx = 0.1m) and vs HR model min, max

Dam-break flow through idealized city (7)



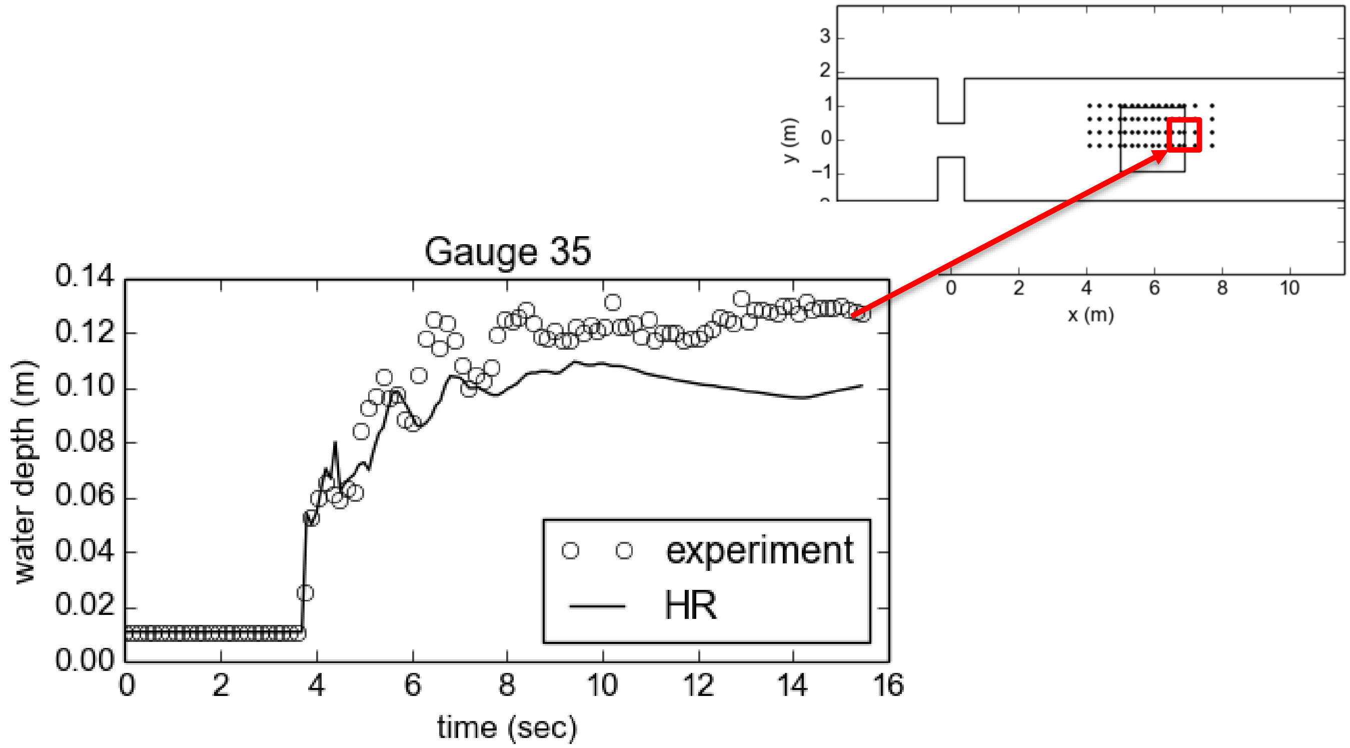
High-resolution model result (HR) vs experimental data

Dam-break flow through idealized city (7)



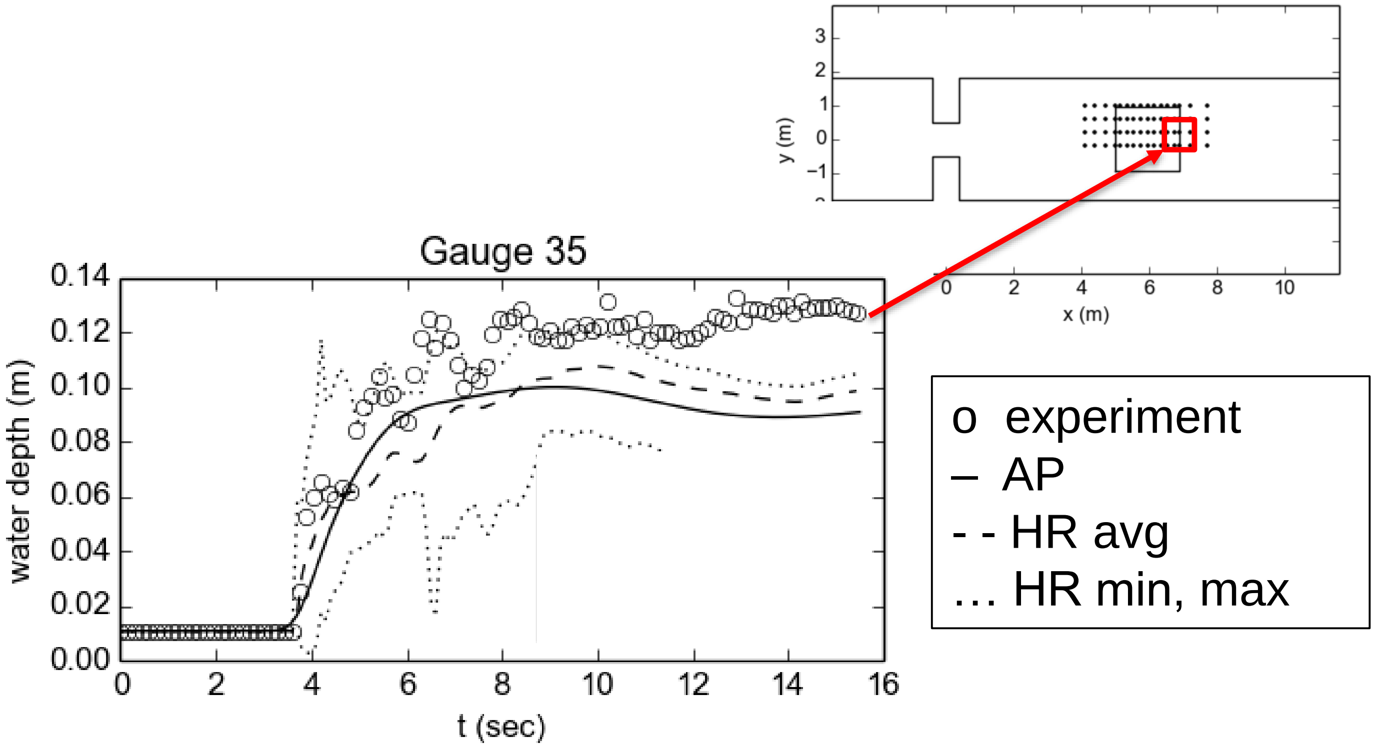
Anisotropic porosity model results (AP, Δx = 0.1m) vs experiment
vs HR model averaged in AP cells (Δx = 0.1m) and vs HR model min, max

Dam-break flow through idealized city (7)



High-resolution model result (HR) vs experimental data

Dam-break flow through idealized city (7)

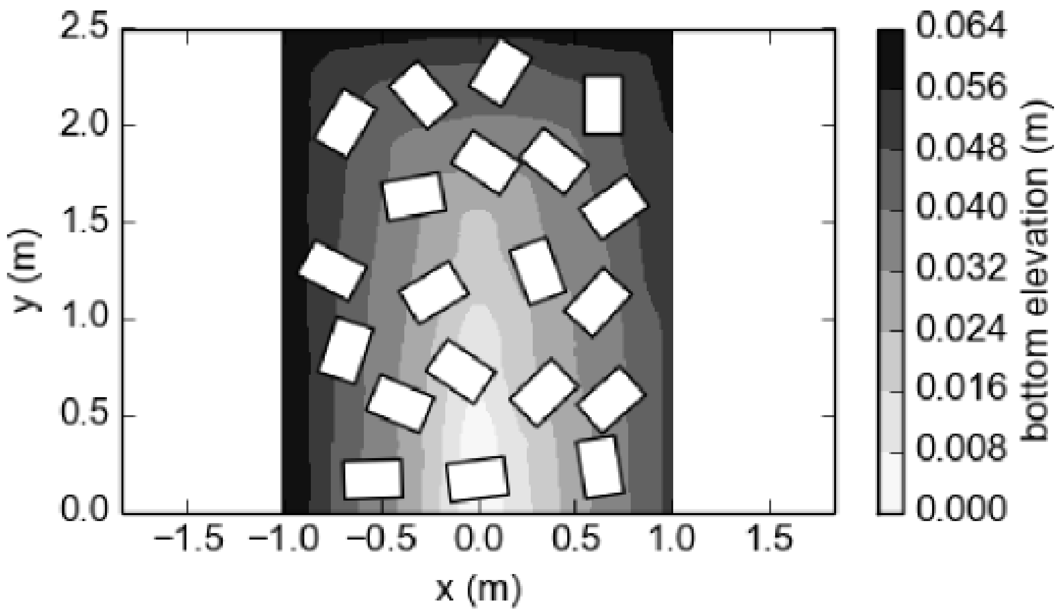


Anisotropic porosity model results (AP, Δx=0.1m) vs experiment
vs HR model averaged in AP cells (Δx=0.1m) and vs HR model min, max

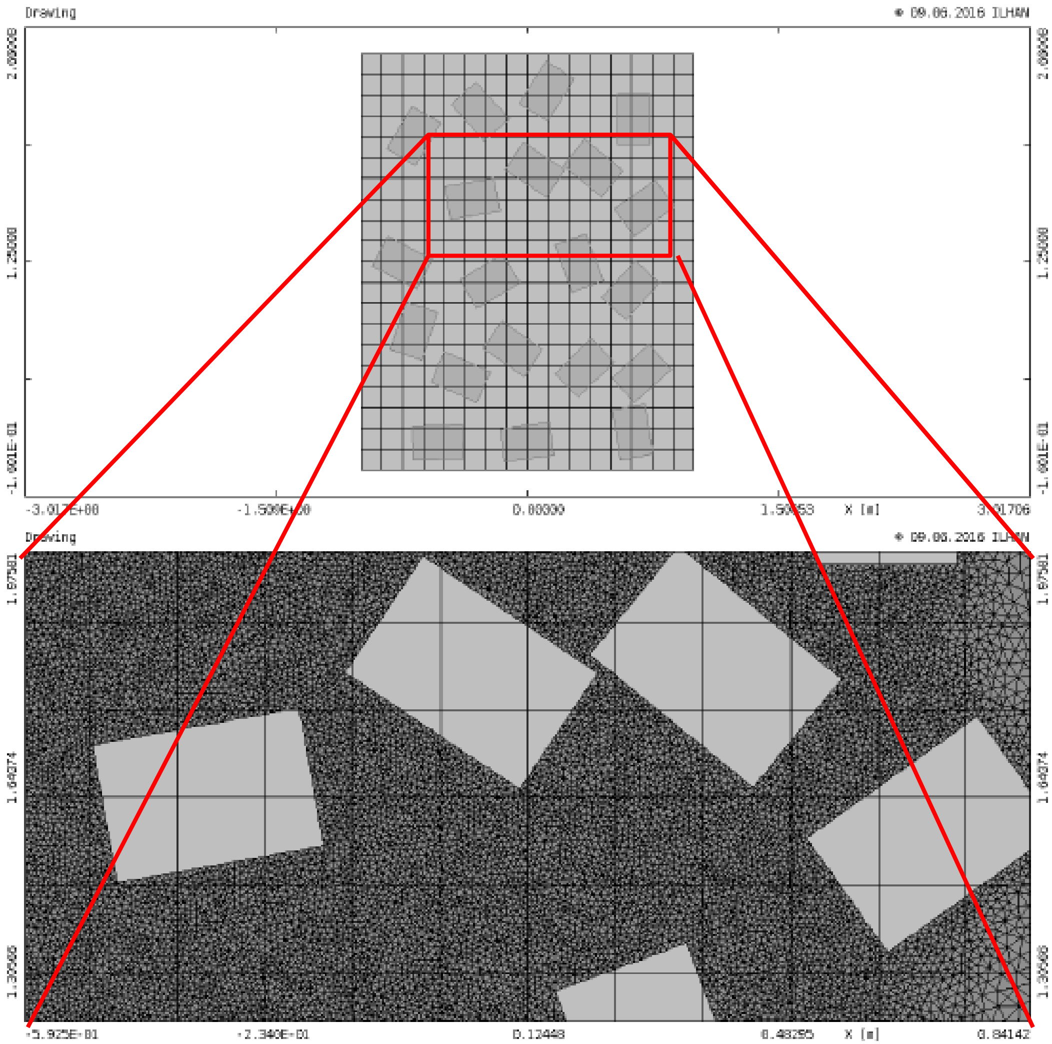
A speedup of about 750 is obtained.

Rainfall-runoff in simplified urban catchment (8)

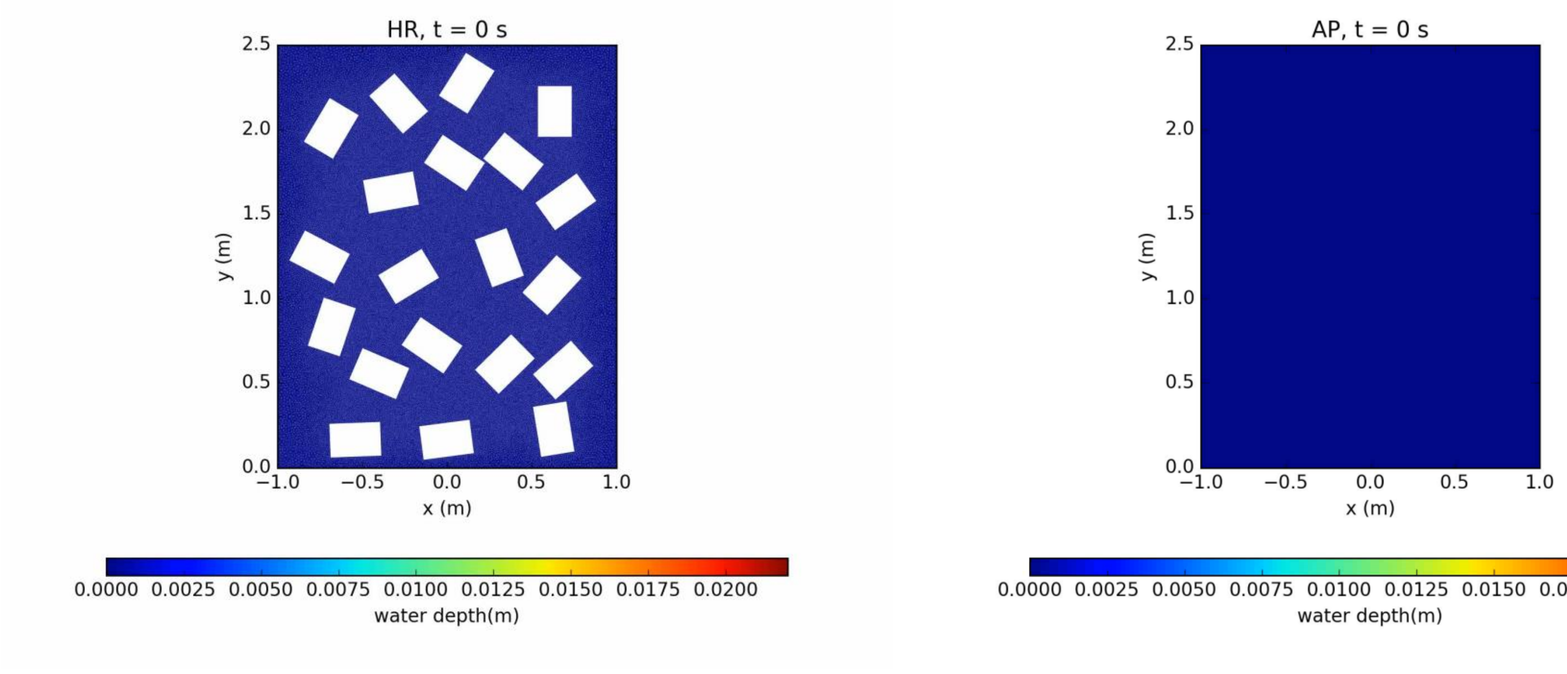
- Laboratory experiment by Cea *et al.* (2010)
- HR model: Δx = 0.01-0.05m, AP model: Δx = 0.125m
- Manning's coefficient: n = 0.016sm$^{-1/3}$
- AP model ac_D = 0.5m^{-1}
- Rainfall intensity: 300mm/h (uniform); duration: 20s



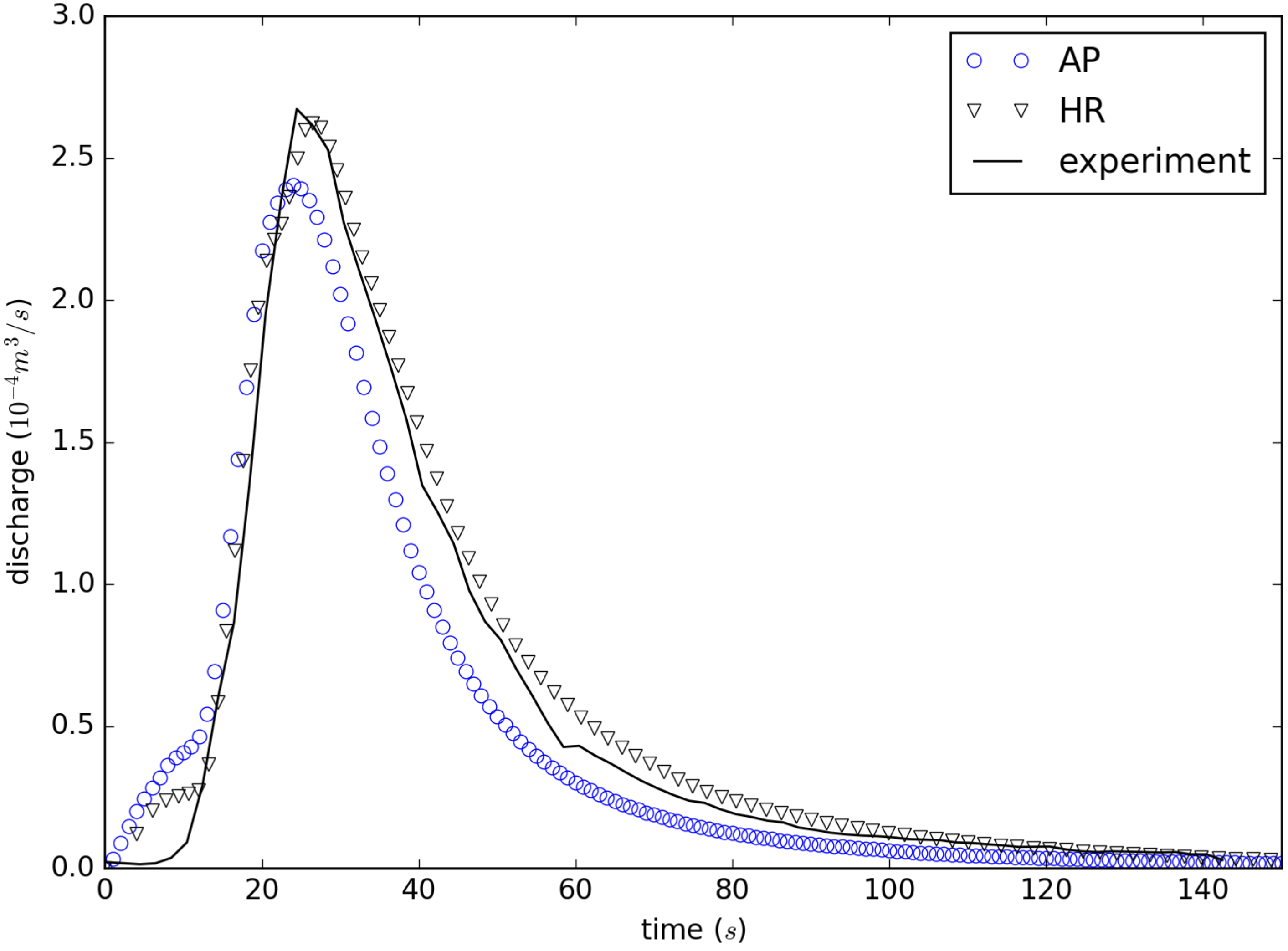
Rainfall-runoff in simplified urban catchment (8)



Rainfall-runoff in simplified urban catchment (8)



Rainfall-runoff in simplified urban catchment (8)



A speedup of about 124 is obtained.

Cell sizes, number of cells, ratios and speedups

Calc.	Size (HR)	Size (AP)	Nr. (HR)	Nr. (CR)	Ratio	Speedup
3	0.01m	0.1m	30 000	300	100	1000
4	0.01m	0.1m	180 000	1800	100	1000
5	0.02m	0.1m	45 000	1800	25	550
6	0.01m	0.4m	28 000	56	500	1140
7	0.01–0.3m	0.25m	96 339	1272	75	750
8	0.01-0.05m	0.125m	62 058	320	190	124

Anisotropic porosity-based upscaling method: Concluding remarks and outlook

- The **anisotropic porosity approach** shows **good agreement** with the high-resolution simulation results for **water levels** and **discharge at outlet**, though *local details below the coarse grid scale* , e.g. discharge in field, can not be resolved.
- **Speedups** between **two and three orders of magnitude** were achieved.
- Good results have been obtained with the **product of** $a \cdot c_D^0 = 10\ \mathrm{m}^{-1}$, based on the authors' experience, the bounds of these product during the calibration should lay in the range [0,20 m^{-1}].
- Özgen *et al.* (2016b) show that the stationary part of the **interfacial pressure term** can be utilized for **well-balance** the numerical model.

Overall conclusions regarding upscaling methods

Overall conclusions regarding upscaling methods for shallow water model

- **Two novel coarse grid upscaling methods** for the fluvial and pluvial flood modeling have been presented and have been applied to academic test cases, laboratoy-scale cases and a “real world“ case.

- The **accuracy of water levels, inundation areas, integral discharges, average flow behaviour and arrival times is satisfying** considering comparisons with high-resolution grids and measurements.
Local flow details, e.g. discharge in field, can not be reproduced, however this can not be expected from such models.

- **Significant speedups of two to three orders of magnitude** were achieved. The **speedup** for both methods (friciton-law based and anisotropic porosity-based) is about the same and **increases with problem size**.

- Overall, they are very suitable for very **fast and real time predictions**.

Overall conclusions regarding upscaling methods for shallow water model

- Overall upscaling methods are an **elegant alternative** to methods of **high-performance computing** (shared-, distributed memory parallel computing, GPU accelerated computing) on high-resolution grids.

- Their **performance can be further increased** by means of **high-performance computing**.

Outlook

- Ongoing work: Improvement on numerical methods for the anisotropic porosity-based method
  - automatic generation of “good“ meshes (gap-conforming meshes)
  - improved flux calculation (collaboration with BTU Cottbus, Germany)
  - real world application case: **flash flood event in Nice** (collaboration with University of Nice Sophia Antipolis, France)
    - Detailed comparison of anisotropic porosity model results with high-resolution results obtained with a parallelized finite-volume code (16h computational time on 128 CPUs) and field measurements

- Scaling approaches could be coupled with **transport processes** (contaminants, heat, sediments/morphodynamics)

- Other possible applications could account for **vegetation** in rivers and reservoirs

- Long term simulations

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Scaling of shallow water models

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END

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